Dynamic Snap-through Buckling of Truss-type Structures

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ABSTRACT: Space trusses under certain conditions may be prone to snap-through buckling. The sudden reduction of the load carrying capacity of a single critical compression member, or a group of compression members, may lead to the snap-through buckling of the compression chord of the structure. This temporary loss of equilibrium due to snap-through normally results in a dynamic force being applied to each node associated with the snap-through. In this paper a methodology based on energy methods is presented to determine the dynamic snap-through response of truss structures. Member failure is taken into consideration by carefully following the buckling load-displacement response of the member. It is assumed that the structure is subjected only to static gravitational loading. A one degree of freedom, simple three bar truss, is used to describe the approach. Finally, this method has been used to perform the snap-through buckling analysis of two double layer grid structure providing a load-displacement behaviour from each structure variation to that obtained from a conventional static analysis.

1. INTRODUCTION

Nowadays, truss systems are widely used for the construction of several types of structures, for example; space structures, long span bridges, transmission towers and offshore platforms. Truss type structures have some major advantages over other structural forms such as a high stiffness, relatively light weight, easy to erect and the ability to cover large open areas. These structures usually have a large degree of static indeterminacy, and therefore it may be expected that after failure of an individual member or a portion of the truss, the remaining structure could carry the redistributed forces and may even carry additional load before collapse. Unfortunately this is not necessarily the case as highlighted by the collapse in 1987 of the double layer grid forming the roof of the Hartford Coliseum[1,2]. The sudden collapse of the Hartford roof has resulted in studies into the stability behaviour of truss-type space structures.

Several factors such as material and connection defects, initial lack of fit, and in particular, buckling phenomena may cause a substantial reduction in member load carrying capacity. Failure of a compression chord member may result in a large redistribution of force within the structure, without any increase in the external load, and consequently if the structure is to remain stable the remaining members have to sustain the additional redistributed forces. One or several of these remaining members may in turn also fail and cause additional force redistribution. In this way, failure can progress through the structure and may ultimately lead to total collapse of the framework. This type of failure mechanism, where an initial local failure is not contained but propagates disproportionately

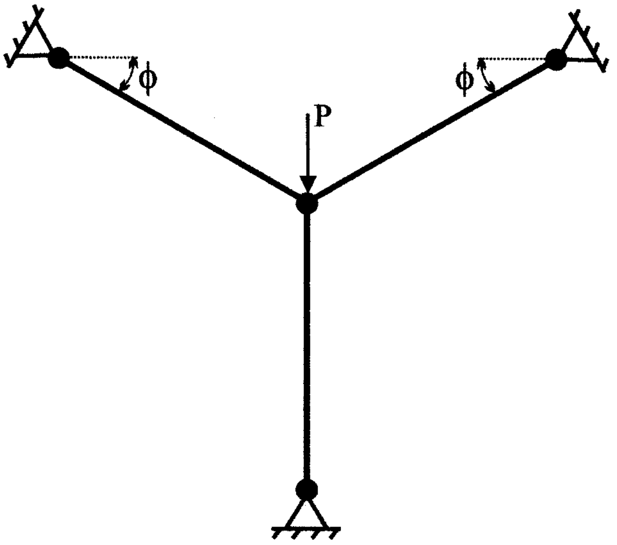


Figure 1. Three bar planar truss. This figure shows a simple plane truss under gravitational load P acting on the central node. All of the members have the same geometrical and material properties and the same length L=2 m. φ=30°.

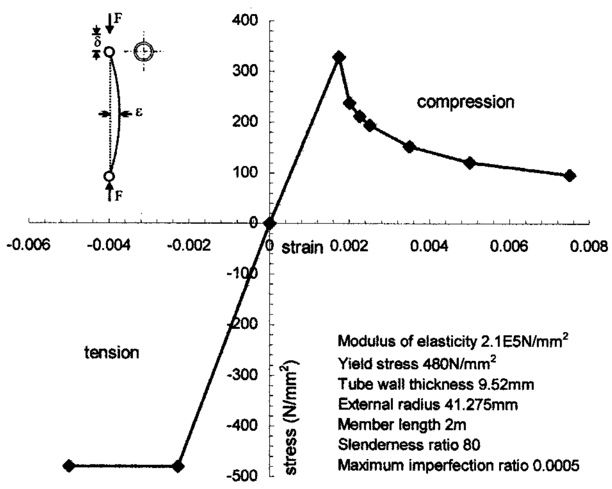


Figure 2. Axial strain-stress relationship of each member of the truss shown in Fig. 1.

throughout the structure, has been termed "progressive collapse". Sudden failure of a member may cause the structure to temporarily loose equilibrium and snap-through into a new equilibrium position. The dynamic effects of the snap-through phenomenon can increase the force redistribution and risk of progressive collapse.

The dynamic effects of snap-through on the response of truss-type structures have been studied by a number of researchers. Using a simple three bar truss (as shown in Fig. 1), Davis and Neal[3,4] showed that this structure will experience the snap-through phenomenon if the absolute value of the negative stiffness of the buckled member is greater than the positive stiffness provided by the rest of the structure. Recently, Morris[5,6] presented a procedure for the ultimate capacity analysis of space trusses whose members have buckled, yielded, or ruptured. The possibility of a dynamic response due to the snap-through behaviour of buckled struts has been considered. Morris pointed out that neglecting the dynamic response due to member snap-through leads to a significant over-estimate of the structure's capacity. Malla et al.[7] and Malla and Nalluri[8] presented a methodology to determine the effects of member failure on the dynamic response of a truss-type space structure. This methodology represented the dynamic failure of a member by the sudden drop in force applied to a joint, equal to the reduction in the member capacity. Also Malla et al.[9] used a pseudo-force method to investigate the progressive failure of a three panel cantilever truss as well as a double layer grid. Abedi and Parke[10] studied the propagation of dynamic, local, nodal snap-through in rigidly-jointed, single-layer braced domes by providing appropriate initial velocities at the nodes in which snap-through occurred.

In this present study, a methodology is presented by the authors to investigate the dynamic effects of member failure on the response of truss-type structures based on energy methods. Member failure is considered by following the buckling response of the member. It is assumed that the structure is subjected only to a gravitational static loading. A one degree of freedom, simple three bar truss, is used to describe the approach. Finally, this method has been used for the snap-through buckling analysis of a double layer grid structure. It should be noted that all of the analysis described in this paper were carried out using the finite element program LUSAS[11].

2. SNAP-THROUGH

Fig. 1 shows a three bar plane truss supporting a gravitation load P. All of the members have the same geometrical and material properties. Fig. 2 shows the axial strain-stress relationship together with the geometrical and material properties used for each member.

It should be noted that in determining the compressive behaviour of the member, shown in Fig. 2, the pin-ended truss member is considered to have a small initial curvature as an imperfection. The maximum initial deviation of the member at mid-point from the chord line was taken to be equal to 0.0005 times the member length (ε = 0.0005L). A member was then modelled in compression using ten elasto-plastic beam finite elements of equal length. Analysing this member, modelled using finite elements, the axial-shortening versus axial-force behaviour of the strut can be determined. The corresponding piece-wise linearised stress-strain relationship is shown in Fig. 2.

The loading P applied to the truss structure shown in Fig. 1 causes a compressive force C in the vertical bar, and a tensile force T in each of the inclined bars. The static vertical equilibrium equation at the loaded node can be written as:

$$P=C+2T\sin\phi \qquad (1)$$

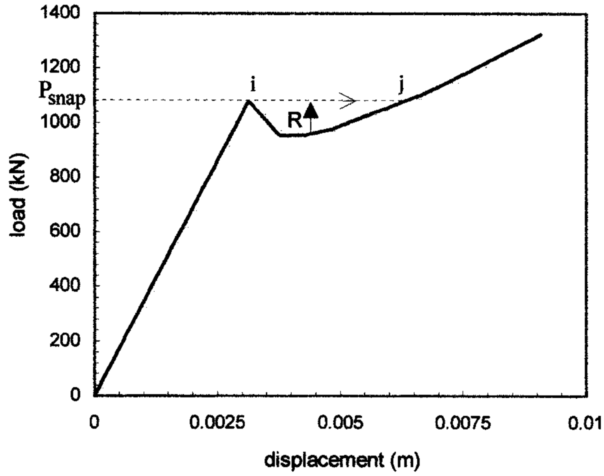
Using Hooke's law and the principle of compatibility of displacements, the tensile force T may be written as a function of the vertical deflection δ of the central node. Accordingly, Eqn. 1 can be expressed as:

$$P=C+\lambda\delta \qquad (2)$$

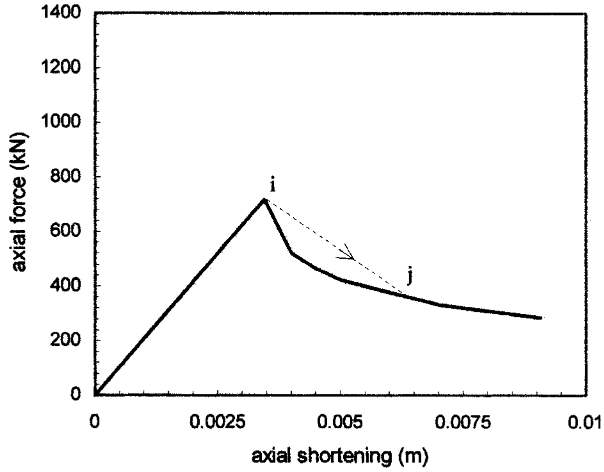
in which,

$$\lambda = \frac{2EA}{L} \sin^2 \phi \qquad (3)$$

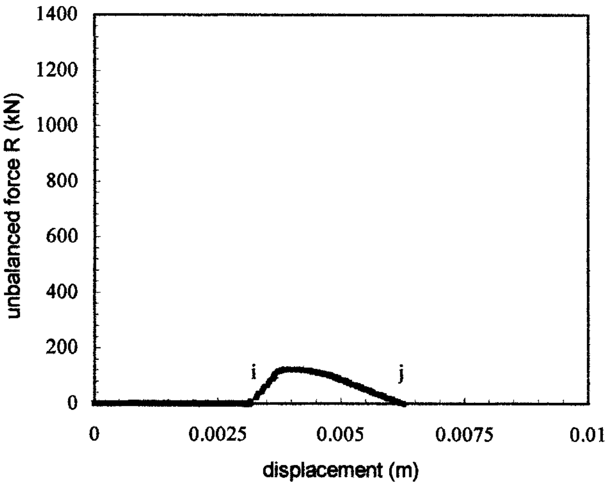
where E, A, and L are Young's Modulus, the cross sectional area and the length of the tensile members, respectively. The parameter λ is the stiffness provided by the tensile members against vertical movement. Eqn. 2 will be valid until yielding of the tensile members occurs, whereupon plastic deformation of the structure occurs. The solid line shown in Fig. 3a represents the static equilibrium path given by Eqn. 2. Buckling of the compressive member occurs at position i, where the applied external load is equal to P_{snap}. As can be seen from the response shown in Fig. 3a, just after buckling of the vertical compression member, equilibrium of the three bar system will be possible only for loading levels below P_{snap}. However, the actual applied loading is a dead load, and hence no load reduction in loading takes place. Thus, buckling of the compressive member leads to a situation in which the structure looks for the nearest equilibrium state, causing a sudden snap-through from position i to position j, (Fig. 3a). From Eqn. 2 the snap-through



(a) Nodal Snap-Through



(b) Member Snap-Through



(c) Unbalanced Load-Displacement Relationship

Figure 3. Dynamic snap-through phenomenon. The figure shows nodal and member snap-through phenomenon along with the unbalanced load distribution.

will happen if $\frac{-dC}{d\delta} > \delta$, that is, the snap-through phenomenon can only occur if the negative post buckling stiffness of the member is greater than the stiffness provided by the rest of the structure. As shown in Fig. 3a, during the snap through from position i to position j, equilibrium is temporarily lost and there would be an unbalanced force R equal to (P_{snap} – P) acting on the mass, m, concentrated at the central node. The unbalanced force R causes the mass, m, to accelerate, giving a dynamic effect to the snap-through phenomenon. Fig. 3c shows the variations in the unbalanced force R with respect to the vertical deflections δ, up to position j. At position j the unbalanced force R reduces to zero, hence the acceleration of the central node will be equal to zero and its velocity will reach a maximum. The velocity of the central node at position j, given by v_j, can be determined using the work-energy equation:

$$U_{i-j} = \frac{1}{2} m(V_j^2 - V_i^2) \qquad (4)$$

in which:

- m is the equivalent mass associated with P_{snap}, and can be expressed as m = P_{snap} /g, where g is the acceleration due to gravity;
- U_{i-j} is the work that has been done on mass m during the transition from position i to position j i.e. kinetic energy gained by the system; this can be obtained as $U_{i-j} = \int_i^j R d\delta$, which is equal to the area of the region bounded by the load-deflection equilibrium path shown in Fig. 3a and the horizontal line drawn between the points i and j;
- V_i is the velocity of mass m at the start of the snap-through (point i in Fig. 3a). Up to this point, no dynamic unbalanced force acts on the structure and hence v_i = 0.

Consequently v_j can be calculated from Eqn. 4 as:

$$v_j = \sqrt{\frac{2U_{i-j}g}{P_{snap}}} \qquad (5)$$

The snap-through from position i to position j happens almost instantaneously. Therefore the dynamic snap-through response of the truss under the constant static load P_{snap} can be determined by applying an initial velocity v_j at the deflected position j of Fig. 3a. The velocity v_j must be applied at the loaded node acting in the direction of the dynamic jump. In this way, the dynamic snap-through response of the truss shown in Fig. 1 can be followed. The time-displacement and time-velocity response for the central loaded node are given in Figs. 4a and 4b. As can be seen from these figures, the damping properties of the structural system cause a gradual reduction in the dynamic vibration. After progressive damping of the vibration, the structure behaves statically and can support further increase in load finally resulting in the collapse of the structure. In this way, the load-displacement behaviour of the central loaded node will be as shown in Fig. 5. This figure also shows a

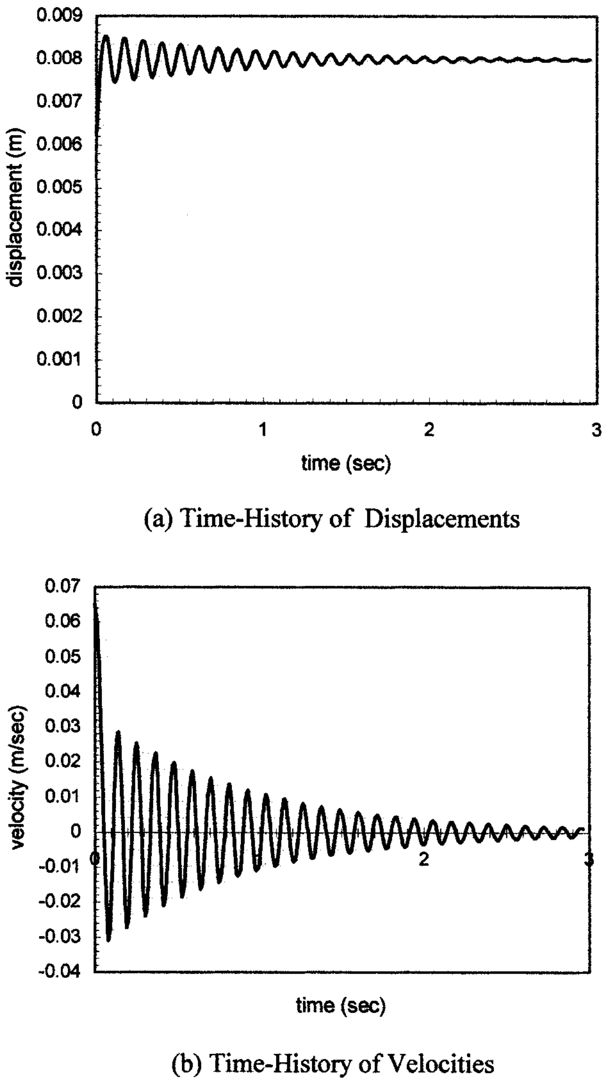


Figure 4. Dynamic snap-through response of the central loaded node of the three bar truss shown in Fig. 1. The truss is subjected to the constant static load Psnap=1076547 N and the initial velocity of the central node v_j=0.0653 m/s at the deflected position j. The damping ratio was taken to be 2%.

comparison of the results of the static analysis with those obtained from the static-dynamic method presented in this paper.

3. FURTHER DISCUSSION ON SNAP-THROUGH PARAMETERS

Kinetic Energy

A truss-type structure may experience the snap-through phenomenon when one or a number of members buckle or loose their load carrying capacity because of other reasons. Because the design loading on the structure is constant at the time of snap-through, the load-displacement behaviour of the nodes and elements of the structure do not follow the unstable equilibrium path shown by the solid line given in Figs. 3a and 3b. Instead the behaviour will follow the straight line between positions i and j shown by the dotted line in Figs. 3a and 3b for instance for the truss given in Fig. 1. Accordingly the kinetic energy gained by the system during the snap-through phenomenon can be expressed in one of the two following forms:

a) Kinetic energy due to nodal snap-through for each loaded node; such as the area of the region bounded by the load-deflection equilibrium path shown in Fig. 3a and the horizontal line drawn between the points i and j.
b) Kinetic energy due to member snap-through for each buckled member; such as the area of the region bounded by the load-deflection equilibrium path shown in Fig. 3b and the straight line drawn between the points i and j.

In a snap-through, the external work of each applied load includes the above mentioned kinetic energy due to nodal snap-through in addition to the conventional summation of strain energy and dissipation of energy due to plastic deformation, i.e. the area under the load-deflection equilibrium path shown in Fig. 3a. Also, the internal energy released from each buckled member includes the above mentioned kinetic energy due to member snap-through along with the conventional summation of strain energy and dissipation of energy due to plastic deformation, i.e. the area under the load-deflection equilibrium path of the member as shown in Fig. 3b. According to the law of conservation of energy, external and internal work must always be equal. Therefore, this law will be satisfied at position j (Fig. 3) of the snap-through, only if the total kinetic energy due to nodal snap-through is equal to the total kinetic energy due to member snap-through. Indeed, these are equivalent terms, and this important concept is used when determining the dynamic snap-through response of the structure, where the dynamic effects of only one of these terms must be applied to the structure.

In the present study, where the snap-through dynamic response of truss structures has been investigated, at first the kinetic energy due to the nodal snap-through in each loaded node has been determined according to a static non-linear buckling analysis, then the initial velocities equivalent to this energy release have been calculated and applied to the corresponding nodes.

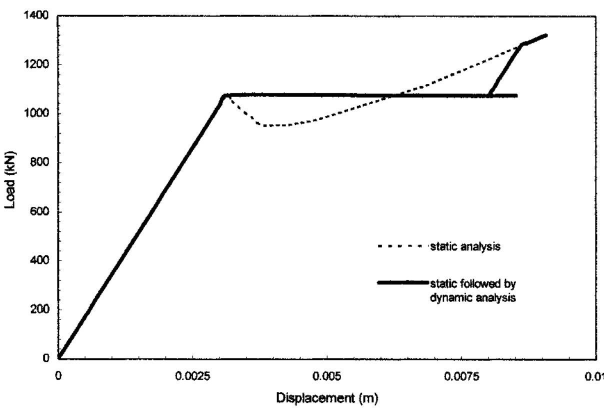


Figure 5. Load-displacement response of the central node of the three bar truss shown in Fig. 1.
This figure shows the comparison of the results of the static snap-through analysis with the results obtained from the proposed static-dynamic method.

Mass

The mass of the structure is an important parameter in dynamic analysis. In the case of gravitational dead loading, the mass of the structure will vary depending on the level of the applied loads. The mass of a structure consists of two portions, the first one is the mass due to the self weight of the structure which is usually small compared to the equivalent mass associated with the applied loads. In the dynamic snap-through analysis special attention must be paid to the mass of the structural loading. As shown in Fig. 6, ignoring the mass of the applied loads will result in an incorrect appraisal of the dynamic snap-through response for the structure.

4. APPLICATION

Fig. 7 shows the geometry and dimensions of a five by five square off set double layer grid, simply supported on roller bearings at the four lower corner nodes.

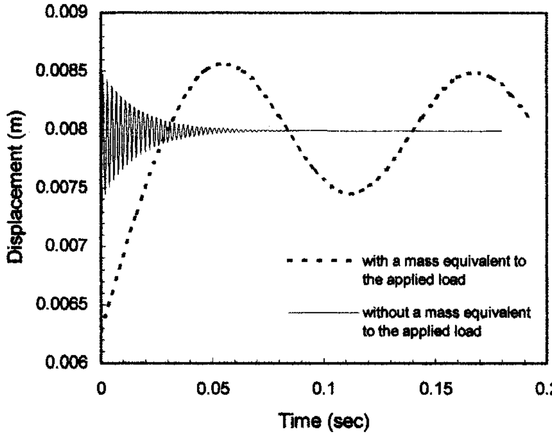


Figure 6. Time-displacement response of the central node of the three bar truss shown in Fig. 1. The figure shows the time-displacement response of the central node of the truss shown in Fig. 1 for two different cases i.e. with and without the equivalent mass of the applied load.

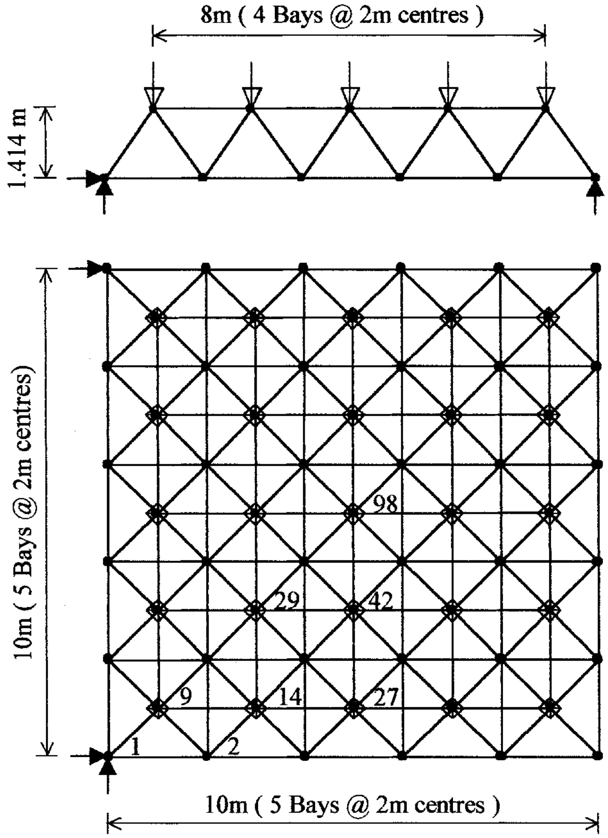


Figure 7. Elevation and plan view of a double layer grid structure supported at the four corner nodes.

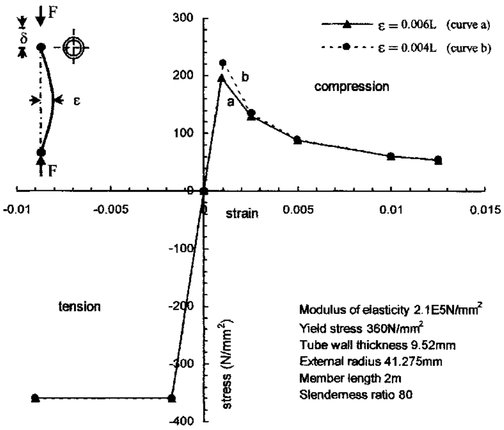


Figure 8. Axial strain-stress relationship for members of the double layer grid shown in Fig. 7.

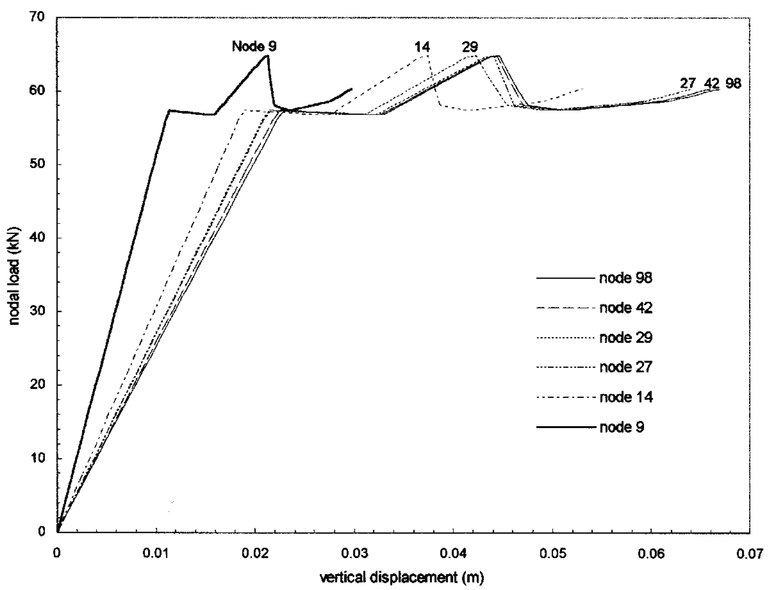


Figure 9. The static load-deflection response of the six upper layer nodes of the double layer grid shown in Fig. 7, ε=0.006L for each member.

Members are assumed to be pinended and made of the same circular tube cross section with an initial lateral imperfection at their centre equal to 0.6 percent of the member length (ε = 0.006L). This is a typical value of the imperfection used to illustrate the snap-through phenomenon in the double layer grid structure shown in Fig. 7. Curve a in Fig. 8 shows the axial strain-stress relationship together with the geometrical and material properties of each member. As an exception, the four diagonal members over the support nodes are considered to be solid bars with same material and geometrical properties as the other members. This strengthening prevents the occurrence of local failure under high shear load over the supports. The structure is subjected to concentrated loads P applied vertically downward to each node of the upper layer.

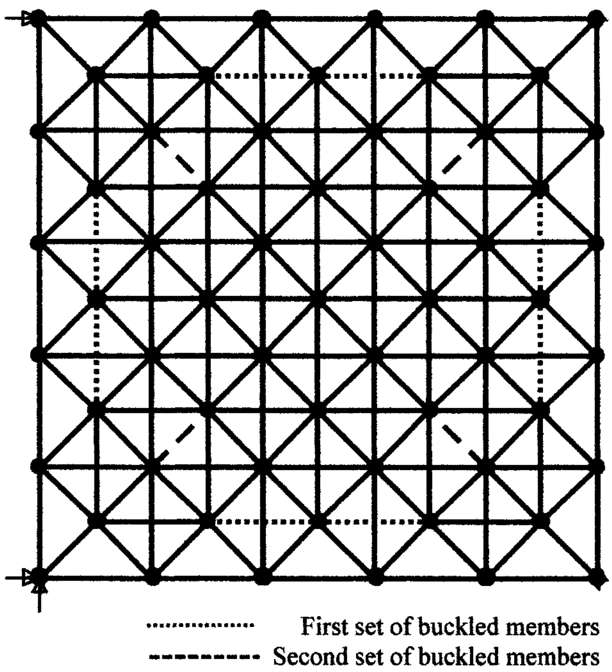


Figure 10. Collapse configuration of the double layer grid structure shown in Fig. 7.

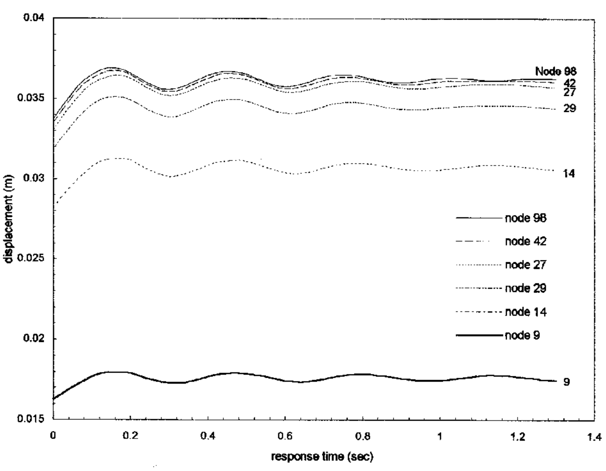


Figure 11. The time-history of displacements for six upper layer nodes of the double layer grid shown in Fig. 7, ε = 0.006L for each member.

Fig. 9 shows the static load-deflection response of six upper layer nodes of the structure. Thus, according to existing symmetry, the load-deflection behaviour of each one of the upper layer nodes will be known. As can be seen, buckling of the first set of members (shown in Fig. 10) occurs at a nodal load level of P_{snap} = 57.4 kN (total load 1.43 MN) which will result in a snap-through. From the static analysis, it is apparent that the maximum nodal load that the structure can support is P_{max} = 64.8 kN and upon reaching this value buckling of a second set of compressive members occurs (also shown in Fig. 10) resulting in the overall collapse of the structure. As stated before, the kinetic energy released at each loaded node, U_{i-j}, can be determined by evaluating the corresponding areas in Fig. 9. Also, the velocity of each loaded node at the deflected position j, v_j, can be calculated using Eqn. 5. The corresponding results are given in Table 1 for the six aforementioned typical nodes. When determining the dynamic snap through response of the structure, the equivalent mass P_{snap} / g = 5848 kg is lumped at each one of the loaded nodes, and the structure at the prescribed deflected position j is subjected to initial velocities v_j as well as the static load P_{snap} = 57.4 kN at each upper layer loaded node. The time-displacement response of the loaded nodes are given in Fig. 11. As can be seen from Fig. 11 the dynamic snap-through causes an oscillation response of the structure, and owing to the damping properties of structural system this oscillation gradually reduced. Consequently as shown in Fig. 12, the final equilibrium state of the structure under the applied load P_{snap} will vary considerably when compared with

Table 1. Kinetic energy released and the corresponding equivalent velocities for six upper layer nodes of the double layer grid shown in Fig. 7. ε = 0.006L for each member.

Node No.	U_{i-j} (N.m)	v_j (m/s)
9	1.720	0.0242
14	3.239	0.0333
27	4.002	0.0370
29	3.636	0.0353
42	3.833	0.0362
98	3.768	0.0359

the behaviour obtained from the static analysis. Fig. 12 also shows that, this structure can be overloaded statically up to overall collapse of the structure.

In order to obtain the response from a similar structure but with a different level of imperfection the grid shown in Fig. 7 was used but the initial imperfection for each member of the structure was reduced to 0.4 percent of its length (ε = 0.004L). The axial strain-stress relationship represented by curve b in Fig. 8 shows the response for a typical member. With this level of imperfection, it can be seen that for this case the negative post-buckling stiffness of the member has increased by 38.9% compared to the former case, hence buckling of a compression member causes a greater force redistribution.

Snap-through occurred in the structure at the load level P_{snap} = 64.4 kN, applied at each top chord node, due to buckling of the first set of members. The static load-deflection response of the six upper layer nodes are shown in fig. 13. The dynamic and the overall collapse load of the structure was P_{max} = 68.9 kN. The position and arrangement of the first and second sets of buckled members are identical to the previous example given in Fig. 10. The released kinetic energy at each loaded node, U_{i-j}, and the corresponding velocity at the deflected position j, namely v_j, are given in Table 2. The equivalent mass of each applied load, used for the dynamic snap-through analysis is P_{snap} / g = 6565 kg.

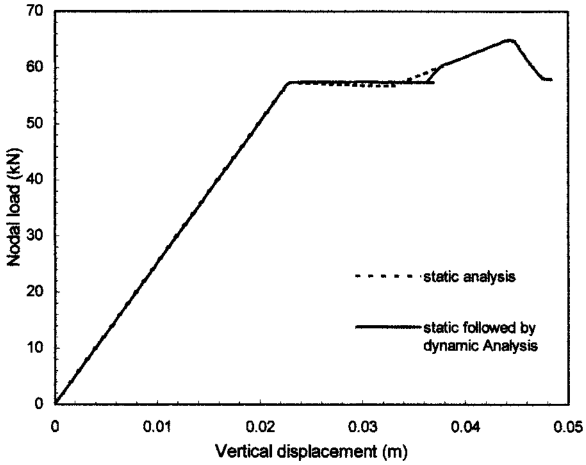


Figure 12. Load-displacement response of the central upper layer node of the double layer grid shown in Fig. 7, ε = 0.006L for each member.
This figure shows the results of the static snap-through analysis oompared with the results obtained from the static-dynamic analysis proposed in this paper.

Table 2. Kinetic energy released and the corresponding equivalent velocities for six upper layer nodes of the double layer grid shown in Fig. 7. ε=0.004L for each member.

Node No.	U_{i-j} (N.m)	v_j (m/s)
9	24.4782	0.0863
14	46.076	0.1184
27	56.903	0.1316
29	51.732	0.1255
42	54.532	0.1288
98	53.618	0.1278

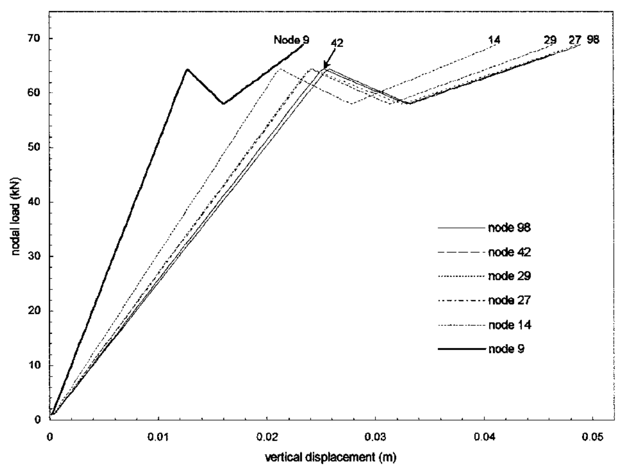


Figure 13. The static load-deflection response of the six upper layer nodes of the double layer grid shown in Fig. 7, ε = 0.004L for each member.

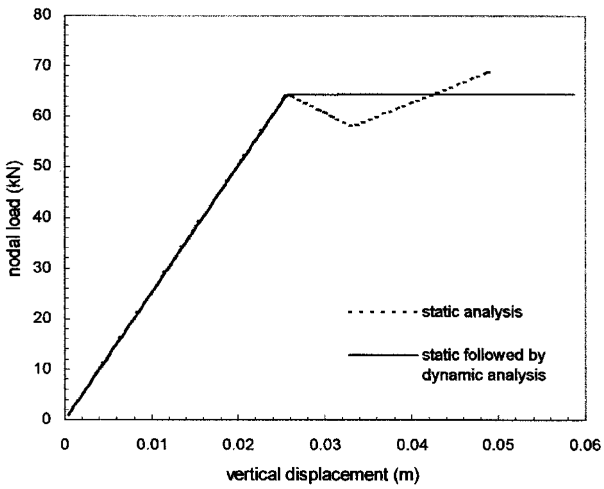


Figure 14. Load-displacement response of the central upper layer node of the double layer grid shown in Fig. 7, ε = 0.004L for each member.
This figure shows the results of the static snap-through analysis compared with the results obtained from the static-dynamic analysis proposed in this paper.

By performing a dynamic snap-through analysis, it is known that the dynamic effects of the snap-through phenomenon cause instability of the structure. The load-displacement behaviour of the central upper node of the structure is shown in fig. 14 and it is compared with the behaviour obtained from the static analysis. As a result, taking into consideration the dynamic effects of the snap-through has led to a 6.5% reduction in the maximum load carrying capacity of the structure compared to the maximum value obtained from a static analysis of the structure. The magnitude of the reduction in load carrying capacity is unique to each structure and load case under consideration and will depend on a range of parameters such as the support conditions and the level of imperfection assumed for the compression members.

5. CONCLUSION

A methodology has been presented capable of evaluating the dynamic effects of member snap-through buckling occurring in the response of truss structures. This has been achieved, by applying to each node involved in the dynamic jump, an initial velocity equivalent to the released kinetic energy. In order to determine correctly the dynamic snap-through response of the structures, a mass equivalent to the applied loads must be considered in the dynamic analysis. Modelling the dynamic effects of member snap-through may lead to collapse of additional members and even the occurrence of progressive collapse. This response causes a reduction in the maximum load carrying capacity of structure compared to that obtained from a conventional static non-linear buckling analysis. Consequently for double-layer trusses prone to snap-through the load-displacement response obtained from a static non-linear buckling analysis may over estimate the maximum load carrying capacity of the structure.

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