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Creation of entangled W states of four two-level atoms in a cavity via quadrapod adiabatic passage

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Abstract In this paper, we considered four two-level atoms coupled with a microwave cavity via stimulated Raman adiabatic passage in quadrapod linkage pattern. Engineering Rabi frequencies of a system with suitable pulse orders, results in entangled W states of four atoms. We also compared the theoretical results with the numerical solutions of the Schrödinger equation in the adiabatic limit which shows the creation of W states at the end of dynamics.

Keywords Quadrapod adiabatic passage · STIRAP · Geometrical phase · Adiabatic approximation

1 Introduction

In the past few years, several quantum coherence and interference effects have been studied theoretically and experimentally (Scully and Zubairy 1997; Ficek and Swain 2004). Some of these phenomena are reservoir induced transparency (Paspalakis et al. 1999; Angelakis et al. 2000), electromagnetically induced transparency (Harris et al. 1990; Boller et al. 1991), EIT-enhanced four-wave mixing (Wu et al. 2003; Deng and Payne 2003), gain without inversion (Imamoglu and Ram 1994; Kitching and Hollberg 1999), coherent population trapping (Vanier et al. 1998; Michaelis et al. 2006), and so on.

Quantum entanglement, one of the most striking features of quantum mechanics, has become an essential element in quantum information processing. Two-mode optical entanglement has already been produced experimentally via an optical parametric oscillator in optical crystals (Ou et al. 1992; Zhang et al. 2000; Villar et al. 2005) and four-wave mixing (Ding et al. 2011; Pooser et al. 2009). Stimulated Raman adiabatic passage of STIRAP is a high qualified method used to completely control population transformation between levels at several atomic systems (Bergmann et al. 1998; Unanyan et al. 1998; Amniat-Talab et al. 2011).

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In this method, the complete population transfer occurs between the ground states, without populating the intermediate excited state. The ground state has a relatively high lifetime of spontaneous emission, so decoherence effects resulted from spontaneous emission of atoms are negligible and this is the advantage of the stimulated adiabatic passage.

There are two entangled states: the first one is GHZ-state (Greenberger et al. 1989) and the second is W-state (Dur et al. 2000).

One of the most striking features of W quantum state (e.g. entanglement state) is: $(|0,1,0\rangle + |1,0,0\rangle + |0,0,1\rangle)/\sqrt{3}$, If we trace the state of one particle, the state of the system will be remain entangled, or if we measure a particle at $\{|0\rangle, |1\rangle\}$ other particles would be remain in maximally entangled state.

In this paper, we derive the corresponding propagator in the adiabatic limit and show that it allows the generation of any coherent superposition of the ground states in quadrapod system by adiabatic passage in a single step with the use of pulses of Gaussian forms. In Sect. 2, we determine the effective Hamiltonian of the quadrapod system to an effective system. We show that one can then transfer the population to any superposition of four ground states. In Sect. 3 we introduce time evolution. Section 4 provides a summary of the results.

2 Construction of the effective Hamiltonian

In this paper, to understanding of basic concept of the cavity-laser interaction, which is achieved between four atoms by excited states, we have to analyze this coupling by four pulsed fields denoted by. $G_1(t)$, $G_2(t)$, $G_3(t)$ and $G_4(t)$, as shown in Fig. 1.

The initial and final atomic states of the system take the form of:

$$|\Psi(t_i)\rangle = |e, g, g, g, 0\rangle \tag{1}$$

$$\left|\Psi(t_f)\right\rangle = \frac{1}{2}\begin{pmatrix} |e, g, g, g, 0\rangle + |g, e, g, g, 0\rangle \\ + |g, g, e, g, 0\rangle + |g, g, g, e, 0\rangle \end{pmatrix} \tag{2}$$

where, subscribes i and j showing the atomic states of the system at before and after of interaction with pulsed fields, respectively, with e and g being atoms excited and ground states, respectively.

We assume the fields have a Gaussian form as:

$$G_i(t) = G_{i0} \exp\left[\frac{(t-\tau_i)^2}{T_i^2}\right], \quad i = 1, 2, 3, 4 \tag{3}$$

where, τ_i and T_i are the time delay between the pump and stokes pulses, and the pulse duration, respectively.

For four identical atoms, the evolution of the coupled atoms-cavity-laser system is described by the Hamiltonian

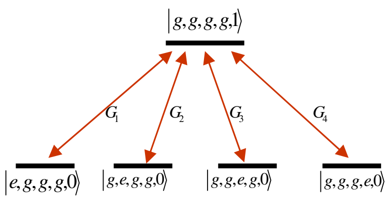

Fig. 1 Linkage pattern coupling four two-level atoms via cavity

$$H(t) = \sum_{k=1}^{4} \left\{ \begin{array}{l} \omega_e \left|e\right\rangle_{k\,k} \left\langle e\right| \\ +[G_k(t)a \left|e\right\rangle_{k\,k} \left\langle g\right| + H.c.] \end{array} \right\} + \omega_c a^+ a \tag{4}$$

where, the summation in the first term describes four atoms and their interaction with the laser and cavity, and the second term describes the cavity energy, with a and a^+ being the annihilation and creation operators for a cavity photon acting upon the states, respectively, and ω_e being the frequency of the coupling excited states.

Assuming:

$$\begin{array}{l} |1\rangle = |e, g, g, g, 0\rangle \\ |2\rangle = |g, e, g, g, 0\rangle \\ |3\rangle = |g, g, e, g, 0\rangle \\ |4\rangle = |g, g, g, e, 0\rangle \\ |5\rangle = |g, g, g, g, 1\rangle \end{array} \tag{5}$$

effective interaction Hamiltonian of the system becomes

$$H^{eff} = \frac{\hbar}{2} \begin{pmatrix} 0 & 0 & 0 & 0 & G_1(t) \\ 0 & 0 & 0 & 0 & G_2(t) \\ 0 & 0 & 0 & 0 & G_3(t) \\ 0 & 0 & 0 & 0 & G_4(t) \\ G_1(t) & G_2(t) & G_3(t) & G_4(t) & 0 \end{pmatrix} \tag{6}$$

Detuning of the frequency has been neglected.

3 Adiabatic time evolution operator

Diagonalizer matrix of the effective Hamiltonian is

$$T(t) = \begin{pmatrix} \frac{G_1 G_2}{\chi_1 \chi_2} & \frac{G_1 G_3}{\chi_2 \chi_3} & \frac{G_1 G_4}{\chi_3 \chi_4} & \frac{1}{\sqrt{2}} \frac{G_1}{\chi_4} & \frac{1}{\sqrt{2}} \frac{G_1}{\chi_4} \\ \frac{-\chi_1}{\chi_2} & \frac{G_2 G_3}{\chi_2 \chi_3} & \frac{G_2 G_4}{\chi_3 \chi_4} & \frac{1}{\sqrt{2}} \frac{G_2}{\chi_4} & \frac{1}{\sqrt{2}} \frac{G_2}{\chi_4} \\ 0 & \frac{-\chi_2}{\chi_3} & \frac{G_3 G_4}{\chi_3 \chi_4} & \frac{1}{\sqrt{2}} \frac{G_3}{\chi_4} & \frac{1}{\sqrt{2}} \frac{G_3}{\chi_4} \\ 0 & 0 & \frac{-\chi_3}{\chi_4} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \frac{G_4}{\chi_4} \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{7}$$

where, we defined

$$\chi_i = \sqrt{G_1^2 + G_2^2 + \cdots + G_i^2} \tag{8}$$

Hence, the Hamiltonian in the basis of adiabatic states can be written as:

$$H^{ad} = T^+ H T - i T^+ \frac{\partial T}{\partial t} \tag{9}$$

In order to obtain the Hamiltonian of the system in terms of adiabatic bases we considered the adiabatic approximation condition as:

$$|\langle d\phi_i(t)/dt | \phi_j(t) \rangle| \leq |E_j - E_i| \tag{10}$$

where, $|\phi_i\rangle$, $|\phi_j\rangle$ are eigenstates of the Hamiltonian and E_i, E_j are eigenvalues of energy. So, we have obtained the adiabatic Hamiltonian of the system as:

$$H^{ad} = \begin{pmatrix} 0 & i\sin\theta_1 & i\sin\theta_2\sin\theta_3\dot{\theta}_1 & 0 & 0 \\ -i\sin\theta_1 & 0 & i\sin\theta_1\dot{\theta}_2 & 0 & 0 \\ -i\sin\theta_2\sin\theta_3\dot{\theta}_1 & -i\sin\theta_1\dot{\theta}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega_k & 0 \\ 0 & 0 & 0 & 0 & \Omega_k \end{pmatrix} \tag{11}$$

where, $\Omega = \chi_4$.

By definition of mixing angles as

$$\tan\theta_i = \frac{G_{i+1}}{\chi_i}, \quad (i = 1, 2, 3) \tag{12}$$

the time evolution operator is given by

$$U^{ad}(t, t_0) = \exp\left(-i\int_{t0}^{t} H^{ad}(t', t_0)dt'\right) \tag{13}$$

Substituting adiabatic Hamiltonian from Eq. 11 into the Eq. 13 gives

$$U^{ad}(t, t_0) = \begin{pmatrix} M & 0 & 0 \\ 0 & e^{-i\delta} & 0 \\ 0 & 0 & e^{+i\delta} \end{pmatrix} \tag{14}$$

where, M is a 3×3 matrix with matrix elements given by

$$M_{ii} = 1 + (Cos\gamma - 1)\left(1 - \frac{\gamma_i^2}{\gamma^2}\right) \tag{15a}$$

$$M_{ij} = \frac{\gamma_i\gamma_j}{\gamma^2}(1 - Cos\gamma) - \varepsilon_{ijk}\frac{\gamma_i}{\gamma}Sin\gamma \tag{15b}$$

where, γ_i can be interpreted as the geometric phase as:

$$\gamma_1(t) = -\int_{t_0}^{t} Sin\theta_3\dot{\theta}_2 dt' \tag{16a}$$

$$\gamma_2(t) = \int_{t_0}^{t} Sin\theta_2 Sin\theta_3\dot{\theta}_1 dt' \tag{16b}$$

$$\gamma_3(t) = -\int_{t_0}^{t} Sin\theta_2\dot{\theta}_1 dt' \tag{16c}$$

$$\gamma^2 = \gamma_1^2 + \gamma_2^2 + \gamma_3^2 \tag{16d}$$

and δ is defined as dynamic phase of the system as:

$$\delta(t) = \frac{1}{2}\int_{t_0}^{t} \Omega(t)dt' \tag{17}$$

Now, by the use of following equations

$$|\Psi(t)\rangle = T(t)U^{ad}(t,t_0)T^{+}(t_0)\,|\psi(t_0)\rangle \quad (18a)$$
$$|\Psi(t)\rangle = U^{eff}(t,t_0)\,|\psi(t_0)\rangle \quad (18b)$$

and by means of the initial state of the system and engineering suitable pulse orders (according to Eq. 2), we can calculate the final state of the system. In the W state, the final state of the system should take the form of

$$\left|\Psi(t_f)\right\rangle = \frac{1}{\sqrt{4}}\left(|1\rangle + |2\rangle + |3\rangle + |4\rangle\right) \quad (19)$$

We have used Gaussian pulses, satisfying the following conditions before and after interaction with the fields, which they are calculated using mixing angles

$$\begin{cases} G_1 = G_m e^{i\pi} \\ T_i = T_m & (m = 2,3,4) \\ \tau_1 = \tau_2 = \tau_3 = \tau_4 \end{cases} \quad (20)$$

$$t \to t_i \Rightarrow \begin{cases} G_1 > G_2 \gg G_3 (= G_4) \\ G_3^2\left(= G_4^2\right) \gg G_1^2\left(= G_2^2\right) \end{cases} \quad (21a)$$
$$t \to t_f \Rightarrow \begin{cases} G_1 > G_3 (= G_4) > G_2 \\ G_1^2\left(= G_2^2\right) \gg G_3^2\left(= G_4^2\right) \end{cases} \quad (21b)$$

So, we can write geometrical phases in Eqs. 16 as:

$$\gamma_1 = \frac{\pi}{4},\, \gamma_2 = \gamma_3 = 0 \quad (22)$$

Then, by substituting results in Eqs. 15, the matrix elements are given

$$\begin{cases} M_{11} = M_{22} = M_{33} \\ M_{12} = M_{13} = 0 \\ M_{32} = M_{23} = -\sqrt{2}/2 \end{cases} \quad (23)$$

At the initial and final interaction times, T(t) can be found by applying pulse conditions as:

$$T(t_i) = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/2 & 1/2 \\ 0 & 0 & -1/\sqrt{2} & 1/2 & 1/2 \\ 0 & 0 & 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \quad (24)$$

$$T(t_f) = \begin{pmatrix} 1/\sqrt{2} & 0 & 0 & 1/2 & 1/2 \\ -1/\sqrt{2} & 0 & 0 & 1/2 & 1/2 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \quad (25)$$

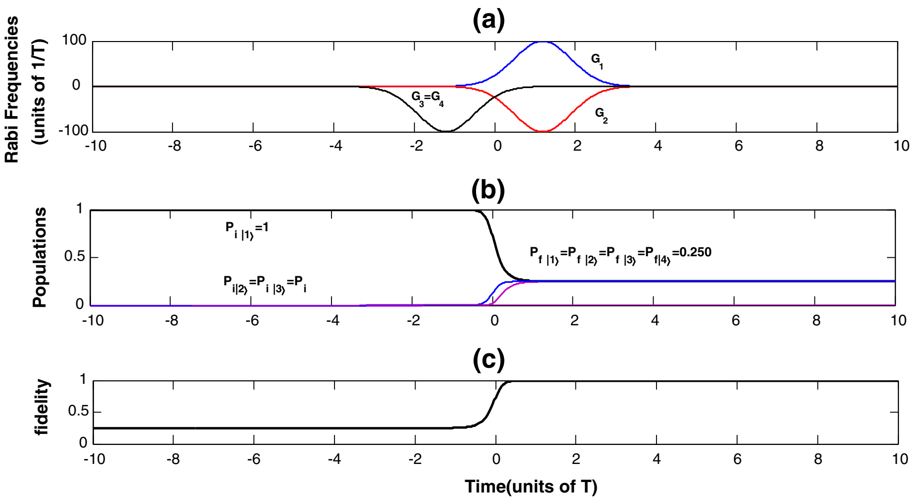

Fig. 2 **a** Plots of Rabi frequencies of the laser fields with the Gaussian pulse parameters of $G_{01} = G_{02} = G_{03} = G_{04} = 40/T$ and with the widths of $T_1 = T_2 = T_3 = T_4 = 1s$ and with $\tau_3 = \tau_4 = 1s$ $\tau_1 = \tau_2 = 0$. **b** Plots the time evolution of the populations. **c** Plots the fidelity of the populations, in different levels as a function of time

Substituting results in Eq. 18, we have

$$|\Psi(t)\rangle = R(t)U^{eff}R^{+}(t)\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix} = \begin{pmatrix}\left[\frac{1}{2}(M_{11}+M_{12})\right]\\ \left[-\frac{1}{2}(M_{11}+M_{12})\right]e^{-i\theta_{21}}\\ \left[-\frac{1}{2}(M_{21}+M_{22})\right]e^{-i\theta_{31}}\\ \left[-\frac{1}{\sqrt{2}}(M_{31}+M_{32})\right]e^{-i\theta_{41}}\\ 0\end{pmatrix} \tag{26}$$

Finally, by substituting results into Eq. 26, the state vector of the system can be found as Eq. 19. The results of numerical calculations are shown in Fig. 2.

4 Conclusion

In conclusion, we have studied a stimulated Raman adiabatic passage in quadrapod linkage pattern driven by three pulses by deriving the corresponding propagator in the adiabatic limit. Engineering Rabi frequencies of the system with a suitable pulse orders allows us to transfer the population to a desired superposition of ground states with final equal populations. It can be seen from Fig. 2c that at final state fidelity of desired state explained Eq. 19 equals to one.

References

Amniat-Talab, M., Saadati-Niari, M., Guérin, S., Nader-Ali, R.: Superposition of states by adiabatic passage in N-pod systems. Phys. Phys. Rev. A **83**, 013817–013817.7 (2011)

Angelakis, D.G., Paspalakis, E., Knight, P.L.: Transient properties of modified reservoir-induced transparency. Phys. Rev. A. **61**, 055802–055805 (2000)

Bergmann, K., Theuer, H., Shore, B.W.: Coherent population transfer among quantum states of atoms and molecules. Rev. Mod. Phys. **70**, 1003–1025 (1998)

Boller, K.J., Imamŏglu, A., Harris, S.E.: Observation of electromagnetically induced transparency. Phys. Rev. Lett. **66**, 2593–2597 (1991)

Deng, L., Payne, M.G.: Inhibiting the onset of the three-photon destructive interference in ultraslow propagation-enhanced four-wave mixing with dual induced transparency. Phys. Rev. Lett. **91**, 243902–243905 (2003)

Ding, C., Li, J., Yang, X.: Controlled optical properties of a five-level atom embedded in photonic crystals with defect modes. Phys. Rev. A **375**, 1737–1745 (2011)

Dur, W., Vidal, G., Cirac, J.I.: Three qubits can be entangled in two inequivalent ways. Phys. Rev. A **62**, 062314–062325 (2000)

Ficek, Z., Swain, S.: Quantum interference and coherence: theory and experiments. In: Springer Series in Optical Sciences, vol. 100, pp. 237–288. Springer, New York (2004)

Greenberger, D.M., Horne, M., Zeilinger, A.: Bell's Theorem, Quantum Theory, and Conceptions of the Universe. Kluwer, Dordrecht (1989)

Harris, S.E., Field, J.E., Imamŏglu, A.: Nonlinear optical processes using electromagnetically induced transparency. Phys. Rev. Lett. **64**, 1107–1110 (1990)

Imamoglu, A., Ram, R.J.: Semiconductor lasers without population inversion. Opt. Lett. **19**, 1744–1746 (1994)

Kitching, J., Hollberg, L.: Interference-induced optical gain without population inversion in cold, trapped atoms. Phys. Rev. A **59**, 4685–4689 (1999)

Michaelis, B., Emary, C., Beenakker, C.W.J.: All-electronic coherent population trapping in quantum dots. Europhys. Lett. **73**, 677–683 (2006)

Ou, Z.Y., Pereira, S.F., Kimble, H.J., Peng, K.C.: Realization of the Einstein-Podolsky-Rosen paradox for continuous variables. Phys. Rev. Lett. **68**, 3663–3666 (1992)

Paspalakis, E., Kylstra, N.J., Knight, P.L.: Transparency near a photonic band edge. Phys. Rev. A **60**, R33–R36 (1999)

Pooser, R.C, Marino, A.M, Boyer, V., Jones, K.M., Lett, P.D.: Low-noise amplification of a continuous variable quantum state. Phys. Rev. Lett. **103**, 010503–010503 (2009)

Scully, M.O., Zubairy, M.S.: Quantum Optics. pp. 220–245. Cambridge University Press, Cambridge (1997)

Unanyan, R.G., Fleischhauer, M., Shore, B.W., Bergmann, K.: Robust creation and phase-sensitive probing of superposition states via stimulated Raman adiabatic passage (STIRAP) with degenerate dark states. Opt. Common. **155**, 144–154 (1998)

Vanier, J., Godone, A., Levi, F.: Coherent population trapping in cesium: dark lines and coherent microwave emission. Phys. Rev. A **58**, 2345–2358 (1998)

Villar, A.S. et al.: Generation of bright two-color continuous variable entanglement. Phys. Rev. Lett. **95**, 243603–243607 (2005)

Wu, Y., Saldana, J., Zhu, Y.F.: Large enhancement of four-wave mixing by suppression of photon absorption from electromagnetically induced transparency. Phys. Rev. A **67**, 013811–013815 (2003)

Zhang, Y. et al.: Experimental generation of bright EPR beams from narrowband nondegenerate optical parametric amplifier. Phys. Rev. A **62**, 023813–023816 (2000)