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Control of instability in a semiconductor laser using a functional pump current generator with a dynamical parameter

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ABSTRACT

In this paper, an electric circuit to control the dynamic output of a semiconductor laser is introduced. The circuit controls chaos and instability of the laser output by changing its pumping current. The change of the current is also introduced by a nonlinear map. The most important element of this nonlinear map is a dynamical variable parameter. We have studied the dynamic behavior of the laser before and after applying the control using bifurcation curves and time series. We have shown that the laser output, in the intervals of the feedback phase and strength where it is chaotic, can be totally inverted to the quasi periodic (QP) and period one (P1) oscillation modes, by control method.

1. Introduction

Chaotic optical communication has attracted much attention because of its potential applications in secure communications [1,2]. Semiconductor lasers are the most important source generators in secure optical communications, coherent light sources for technological optical transition, and ultra fast optical processing [3–5]. Nonlinear dynamics of semiconductor lasers have been widely studied because of the important roles they play in conventional optical communications and in chaotic optical communications. Under external perturbations, such as optical feedback, optical injection, or optoelectronic feedback, various nonlinear dynamics and routes to chaos have been observed and investigated [6]. External cavity semiconductor lasers (ECSLs) are an integral part of high speed chaos based communication systems [7]. Different studies have been conducted to characterize long external cavity [8] and short external cavity [9]. Where, the short cavity regime has more advantages as described in [10]. One of the remarkable discoveries of the recent decade is related to the pervasive presence of chaos in multidynamical systems that has led to a new field of research for control of chaos. The Ott, Grebogi, and Yorke method [11] has been successfully demonstrated to convert a chaotic motion to a periodic regular motion [12] in emerging applications such as encoded communications and design of high quality optical communication systems [13]. Chaos control in the lasers has been extensively used in recent years in applications, such as bulk cavity ring oscillator [14], single-mode CO_2 lasers with an electro-optic feedback on the cavity losses [15], and modulating and variation of the pump current [16,17]. The control of multistability using periodic perturbation has been investigated in different theoretical models such as the Henon map [18], laser models [19], coupled Duffing oscillators [20] and a delayed logistic map [21]. In this paper, we have investigated stability of the laser output by controlling parameter 'ε' and pump current dynamically. Dynamical control parameter and pump current have been selected as implementation tools of control method. This method is simple and easy to implement experimentally. The method involves the continuous generation of an error signal from the difference between the output intensity, $|E^2|$, signal and its value at an earlier time. It is called continuous time-delay feedback. This method was useful in fast system applications to control a chaotic nonlinear circuit [22], where a basic role is played by an electrical circuit. The output of laser, after conversion to the photodiode voltage, is sent to a comparator. The comparator injects a suitable pumping current is to the laser via the switches 1 and 2 of the electrical circuit. This current, which is used to stabilize the output of the laser, is generated by the chaotic map. It is noted that the value of the ε can be changed manually by operators. An important advantage of this method is that it is enough to have information only for one variable. The experimental setup is shown in Fig. 1. We have studied the dynamical behavior of the laser as a function of feedback phase and strength, before and after applying the control method, as it is described in this section, using bifurcation curves and time series. It has been shown that in some

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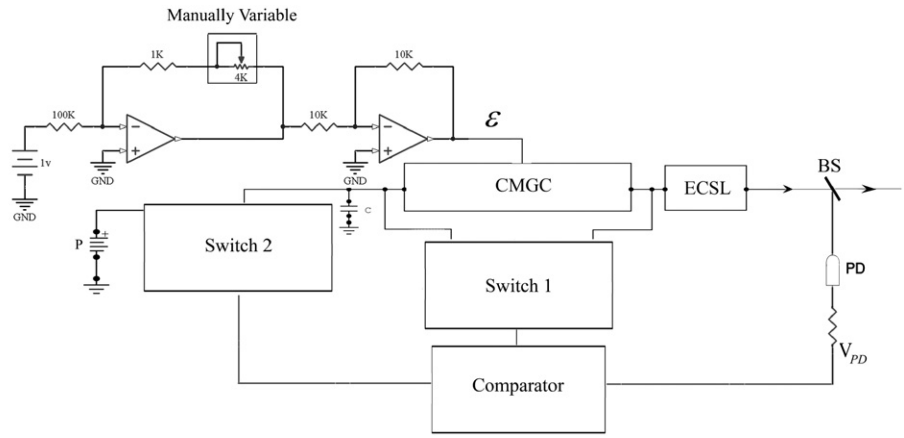

Fig. 1. Optical setup based on semiconductor laser with controller electrical circuit.

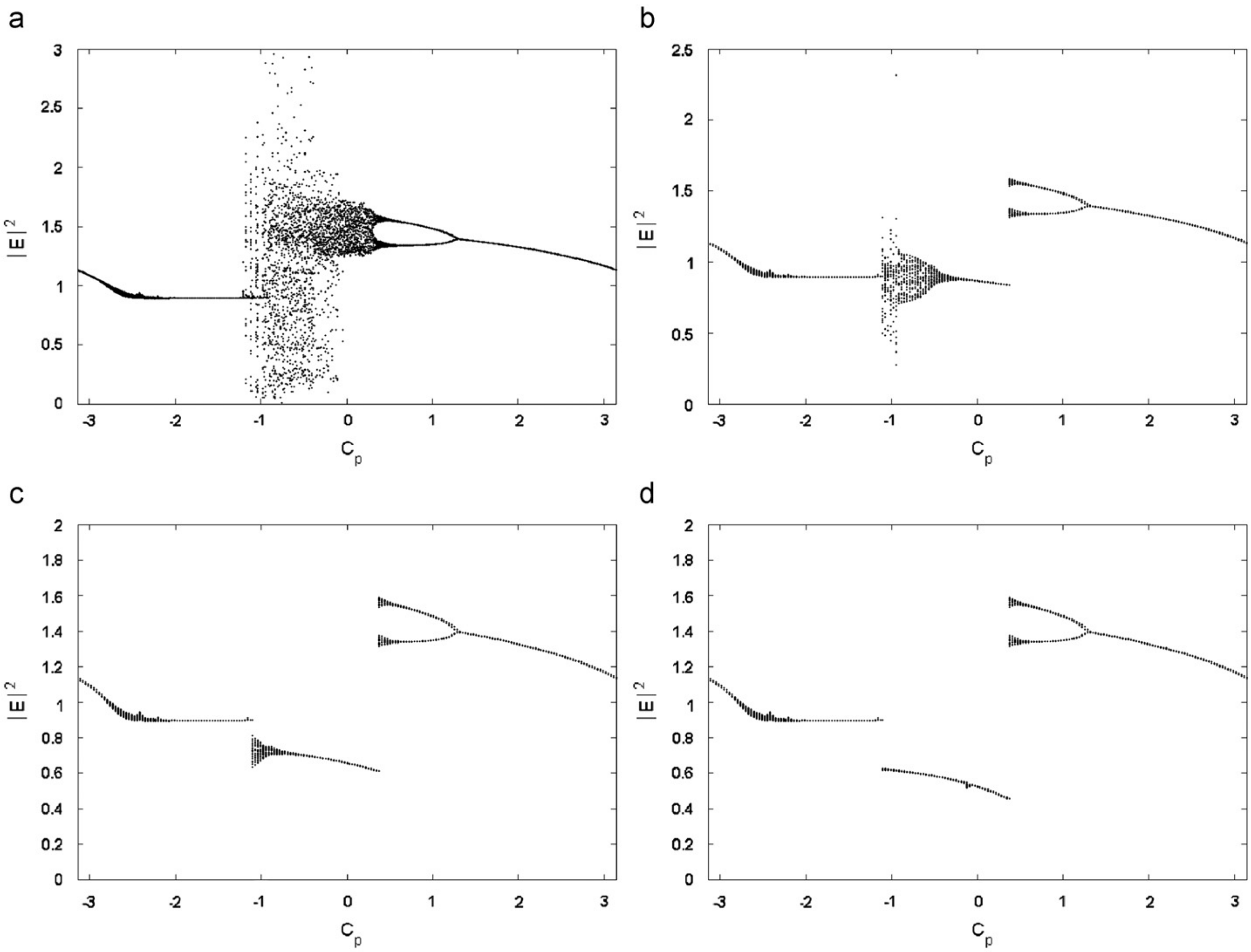

Fig. 2. Bifurcation of $|E^2|$ versus C_p with $\eta=0.036$ and $P=0.8$, before applying control method (a), after applying control method with $\varepsilon=0.01$ (b), $\varepsilon=0.02$ (c), and $\varepsilon=0.03$ (d).

ranges of the feedback phase and strength, the laser output may be chaotic. We showed that the dynamics of the laser output, in these ranges, could be controlled to a stable state by applying the control method. Our studies are theoretical, and for implementation of the control feedback loop experimentally, it is attempted to introduce an optical setup containing electrical circuits.

2. The laser model and the control method

In the early 1980s, Lang-Kobayashi (LK) proposed a model for SCLs [23,24]. The LK equations describing the model were used extensively in the past to describe a semiconductor laser subject to feedback from an external cavity [24]. Where they used delay differential equations with the advantage that have an infinite-dimensional phase space [25,26]. The LK equations can be written in the following dimensionless and compact form [16]:

$$\frac{dE}{dt} = (1+i\alpha)NE + \eta E(t-\tau)e^{-iC_P}, \quad (1)$$

$$T\frac{dN}{dt} = \mathbf{P} - N - (1+2N)|E|^2, \quad (2)$$

$$\boldsymbol{P} = \begin{cases} \frac{1}{\varepsilon^2}\tan^2(2\tan^{-1}(\sqrt{P_m})) & \text{for}(V_{PD} > V_{\max}), \\ P & \text{for}(V_{\min} < V_{PD} < V_{\max}) \text{ or } (V_{PD} < V_{\min}). \end{cases} \quad (3)$$

Eqs. (1), (2) describe a semiconductor laser with external optical feedback. For the complex electric field E and inversion N parameters, α is the line width enhancement factor, C_p is the 2π-periodic feedback phase, $\boldsymbol{P}$ is the pump current and η is the feedback strength. We found that the output dynamic of the laser is chaotic with feedback rate of $\eta=0.0455$ (and $\eta=0.036$) [27], this corresponds to a feedback induced reduction of threshold relative to the solitary laser of ~%7.

In these equations, the time is normalized to the cavity photon lifetime (1 ps) and T is the ratio of the carrier lifetime (1 ns) to the photon lifetime [10]. The external round trip time, τ, is also normalized to the photon lifetime. Different parameters are held fixed at $T=1710$, $\tau=70$, $\alpha=5.0$, $P=0.8$ and 1.1 [9]. Our studies are on a short external cavity in which, the frequency of external cavity substantially exceeds the oscillation frequency of solitary laser relaxation. It should be noted that, the cavity photon life time (1 ps) has only used for rescaling the time, so that a delay time of $\tau=70$ corresponds to an external cavity length of ≈1.1 cm. However, the control feedback loop containing pumping current generator is limited to a characteristic response time of ≈100 ns, which corresponds to a bandwidth of ≈0.01 GHz. When Laser behaves chaotically, without C_p or η being varied, it will remain in the same state. On the other hand, passing of time will not provide any change in the type of the output intensity dynamic, unless they are changed. But, it seems that with pumping current, produced by the control feedback loop that is injected with a time delay of τ to the laser, the instability of the laser output will be also controlled. The simple model for controlling the stability of laser is introduced by Eq. (3) [28,29]. We select the variation of ε to be in the range of 0–0.5, [28]. Different values of the ε in this interval are selected such that with suitable pumping current, the output of the laser

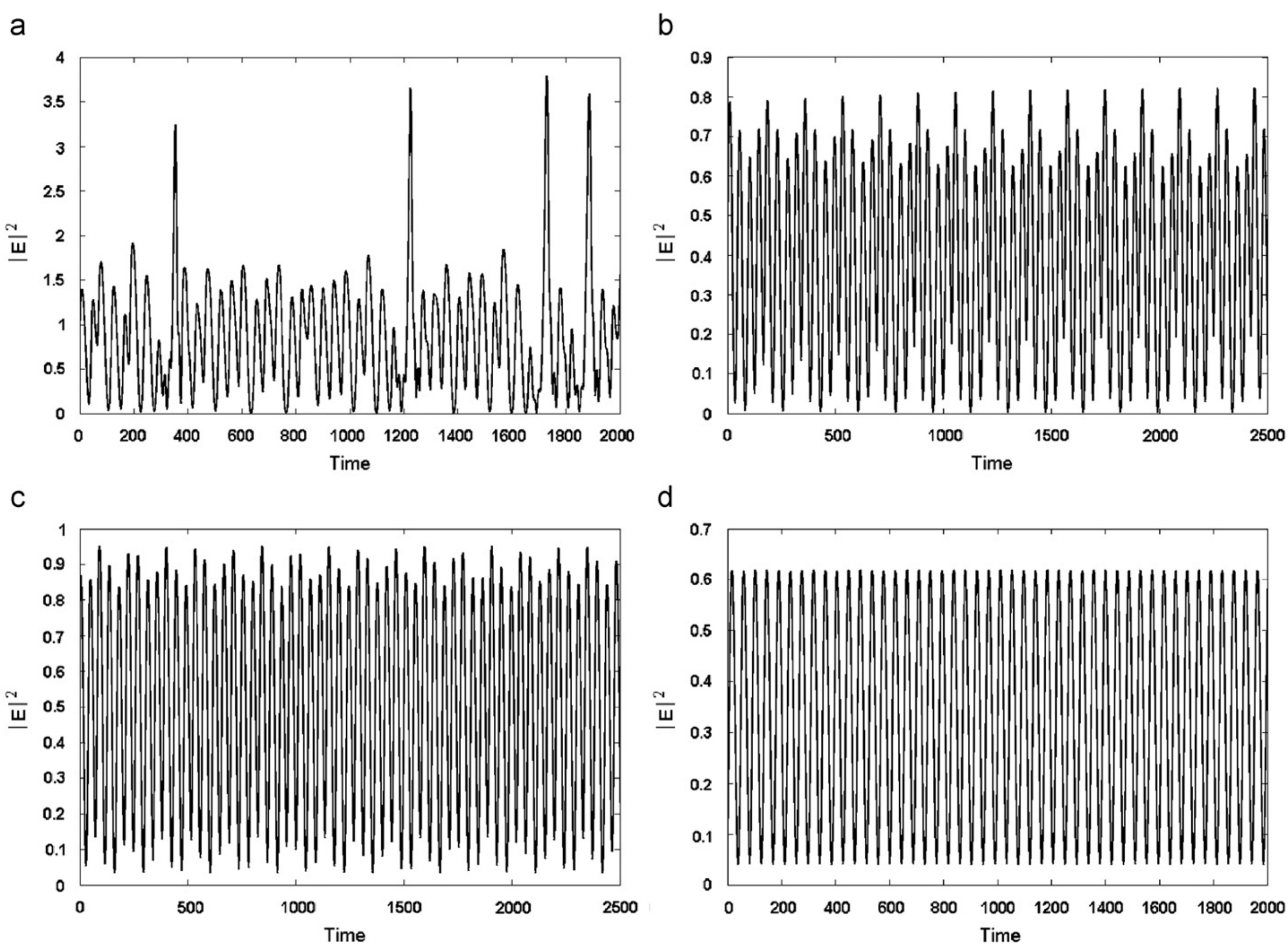

Fig. 3. Time series of $|E^2|$, with $\eta=$ 0.036 and $P=0.8$, before applying control method (a), after applying control method with $\varepsilon=0.01$ and $C_p=0.527$ (b), $\varepsilon=0.02$ and $C_p=1.005$ (c), $\varepsilon=0.03$ and $C_p=1.005$ (d).

reaches a stable state. For continuously operation of Eqs. (1) and (2), constant values of the parameters $\boldsymbol{P}$,T,τ and α are needed. As the laser operates instable, another constant value of parameter $\boldsymbol{P}$ is introduced to the Eqs. (1) and (2) using Eq. (3) and feedback loop.

The setup containing the laser and the controlling circuit is shown in Fig. 1. The most important part of the system is the laser pump current controller. The pump current is controlled by the use of an electrical circuit. The controller uses the switch1, switch 2 and chaotic map generator circuit (CMGC) to produce Eq. (3). Details of CMGC are explained in Appendix A. The circuit also uses a photodiode (PD) to convert a laser light to an electrical current and then using the resistance R to electrical voltage "V_{PD}" which is connected to the comparator. In the stable states, V_{PD} varies between V_{min} and V_{max}. For $V_{PD} < V_{min}$ the laser shuts off, and for $V_{PD} > V_{max}$ the laser operates chaotically. Furthermore, in the stable operation of the laser, the switch1 is ON and switch2 is OFF, and the pump current with constant value of $P=0.8$ (and 1.1) is connected to the input of the laser and the coupled Eqs. (1) and (2) describe a semiconductor laser with external optical feedback. In unstable states, the laser intensity is increased, the voltage V_{PD} becomes more than V_{max}, the switch2 disconnects P from the laser input, and simultaneously switch1 connects the CMGC output to the laser input. During this process, the laser continues its action by initial value of $P=0.8$ (and 1.1), that was stored on the capacitor C.

By receiving the feedback from the output of the laser, P_{m+1} is continually decreased, making the photodiode output voltages to be decreased as well. Finally, when the output voltage of the photodiode becomes less than V_{max}, switch1 is disconnected but switch2 is still on, and the capacitor "C" at the input of CMGC keeps the last P_{m+1}. In fact, when a suitable value of $\boldsymbol{P}$ is generated by CMGC (for stabilizing the operation of the laser), the possibility of continues operation will be provided by means of the capacitor C, by storing different amounts of $\boldsymbol{P}$. As long as the amount of $\boldsymbol{P}$ is changing until the establishment of the condition of $V_{min} < V_{PD} < V_{max}$, the capacitor C plays the role of a current pumping in a continues way. After a while, if the laser intensity further decreases, and "V_{PD}" becomes less than V_{min}, the switch2 becomes off and the P connect to the laser input, and the system changes to normal condition. The circuit also has a manual controller 'ε' which is the second input of the CMGC. The output intensity of the laser, $|E^2|$, and consequently the output current of the PD and V_{PD} can have different values in different oscillation dynamics (P1, QP, chaotic,) of the laser. The $|E^2|$ ($\propto I_{PD}$) and consequently the V_{PD} can be changed by different values of ε. Therefore, in the ranges of feedback phase or feedback strength, in which the output of the laser is chaotic ($V_{PD} > V_{max}$), the ε can be

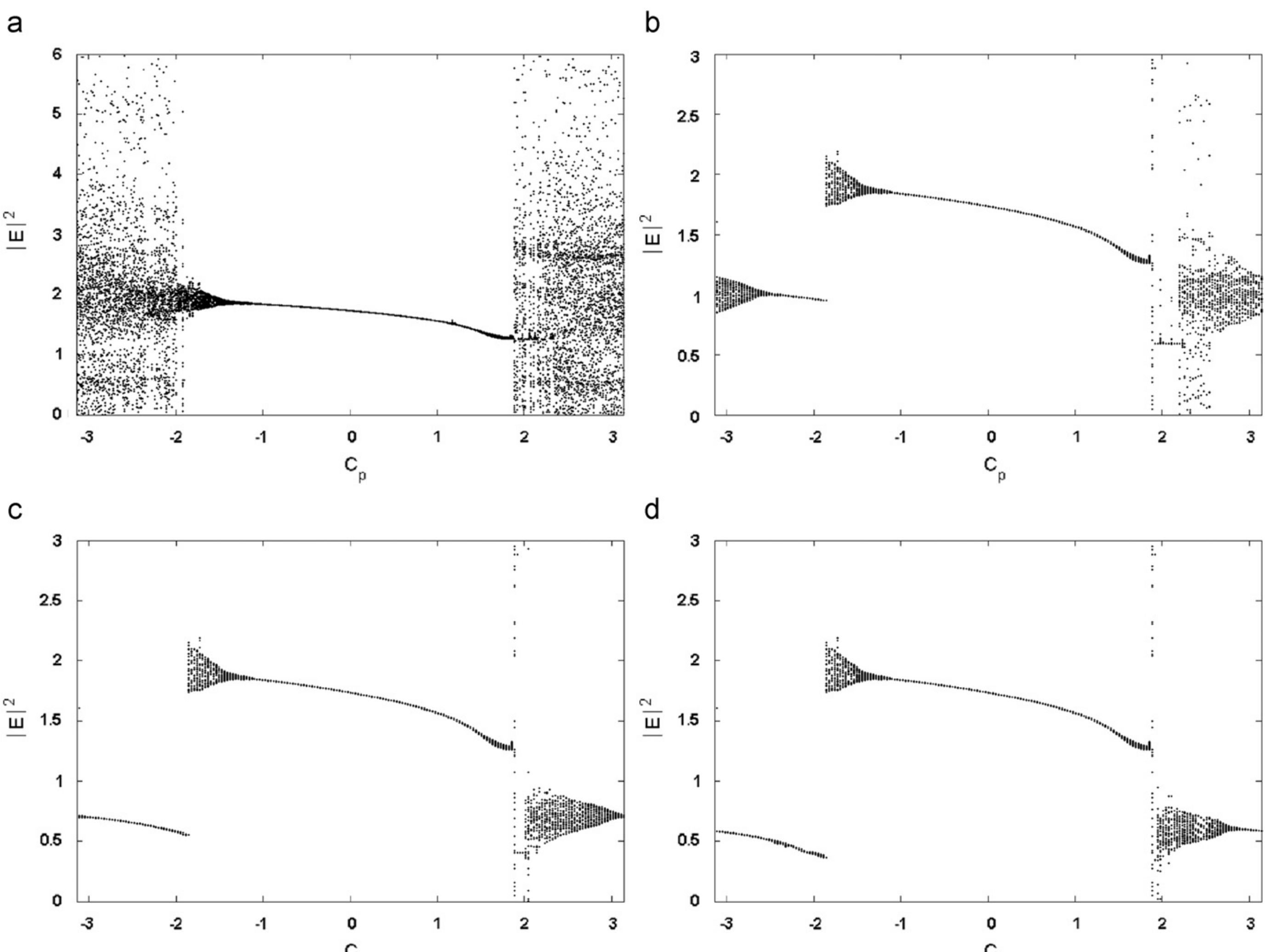

Fig. 4. Bifurcation of $|E^2|$ versus C_p with $\eta=0.0455$ and $P=1.1$, before applying control method (a), after applying control method with $\varepsilon=0.02$ (b), $\varepsilon=0.03$ (c), and $\varepsilon=0.04$ (d).

selected such that the V_{PD} becomes less than V_{max} and the laser operates in the stable mode. For studying output intensity dynamics of the semiconductor laser using the control method mentioned earlier, feedback phase and feedback strength are selected as control parameters. It should be stated that this model holds only for single-mode semiconductor lasers when subject to extremely small optical feedback. This implies that edge-emitting lasers should be avoided, since they can present multi-(longitudinal) mode dynamics in particular in the presence of optical feedback.

3. Feedback phase as a control parameter

In the control method mentioned above, we have used three different values of ε, $\varepsilon = 0.01$, $\varepsilon = 0.02$, and $\varepsilon = 0.03$. Before applying the control, the dynamical behavior of the $|E^2|$ as a function of C_p is shown in Fig. 2a. After applying the control method, the dynamics of the $|E^2|$ as a function of C_p for $\varepsilon = 0.01$, 0.02 and 0.03, are shown in Figs. 2b–d, respectively. For these cases, C_p is used as a control parameter. In Fig. 2a, the feedback strength and the laser pump current have the values of $\eta = 0.036$ and $P = 0.8$, respectively. Fig. 2a shows that for some negative values of the C_p, the $|E^2|$ operates in the periodic dynamics (P1). As the feedback phase is increased, the periodic dynamics undergoes a chaotic domain ($-1.147 < C_p < 0.314$), leading to the QP region. Then, the dynamics of the $|E^2|$ shows an inverse period doubling, leading to period1 (P1). In Figs. 2b–d, η is fixed at 0.036 and $\boldsymbol{P}$ is varied by the control method. In these cases, $\boldsymbol{P}$ becomes 0.08 for stable areas (periodic behaviors), while for chaotic areas of $|E^2|$ as a function of C_p, it is changed to P_{m+1} by the control method. This process is performed by the variation of ε in Eq. (3). With $\varepsilon = 0.01$, the wide range of the chaotic dynamics ($-1.147 < C_p < 0.314$) in Fig. 2a is reduced to a narrow chaotic column ($-1.1 < C_p < -0.942$) and to a tours bifurcation ($-0.911 < C_p < -0.314$) leading to P1, as it can be seen in Fig. 2b. As Fig. 2c shows, with $\varepsilon = 0.02$, the chaotic area in the output of the laser is completely removed. In this case, the range of the QP is decreased, and the range of P1 is increased. Finally, with $\varepsilon = 0.03$, the dynamics of the $|E^2|$ is totally controlled and becomes stable by the control method. In this case a P1 dynamics appears instead of the chaotic one, as it can be seen from Fig. 2d. To validate the above results, we have investigated the time series of the $|E^2|$ for values of the ε mentioned above. The dynamics of the $|E^2|$ in terms of time, for the case before applying the proposed technique, is represented in Fig. 3a, and for $\varepsilon = 0.01$, $\varepsilon = 0.02$ and $\varepsilon = 0.03$ they are depicted in Figs. 3b–d, respectively. The time series appeared in Fig. 3b represents a QP dynamics. Fig. 3c also shows a QP dynamics, but with reduced number of the oscillation modes. This can be observed by changes the height of the peak after the highest peak. Fig. 3d shows a P1 dynamics for $|E^2|$ in terms of time. To study the efficiency of the control method further, we tested it for $\eta = 0.0455$ and $P = 1.1$. Fig. 4d shows the dynamics of the laser, $|E^2|$, as a function of C_p before

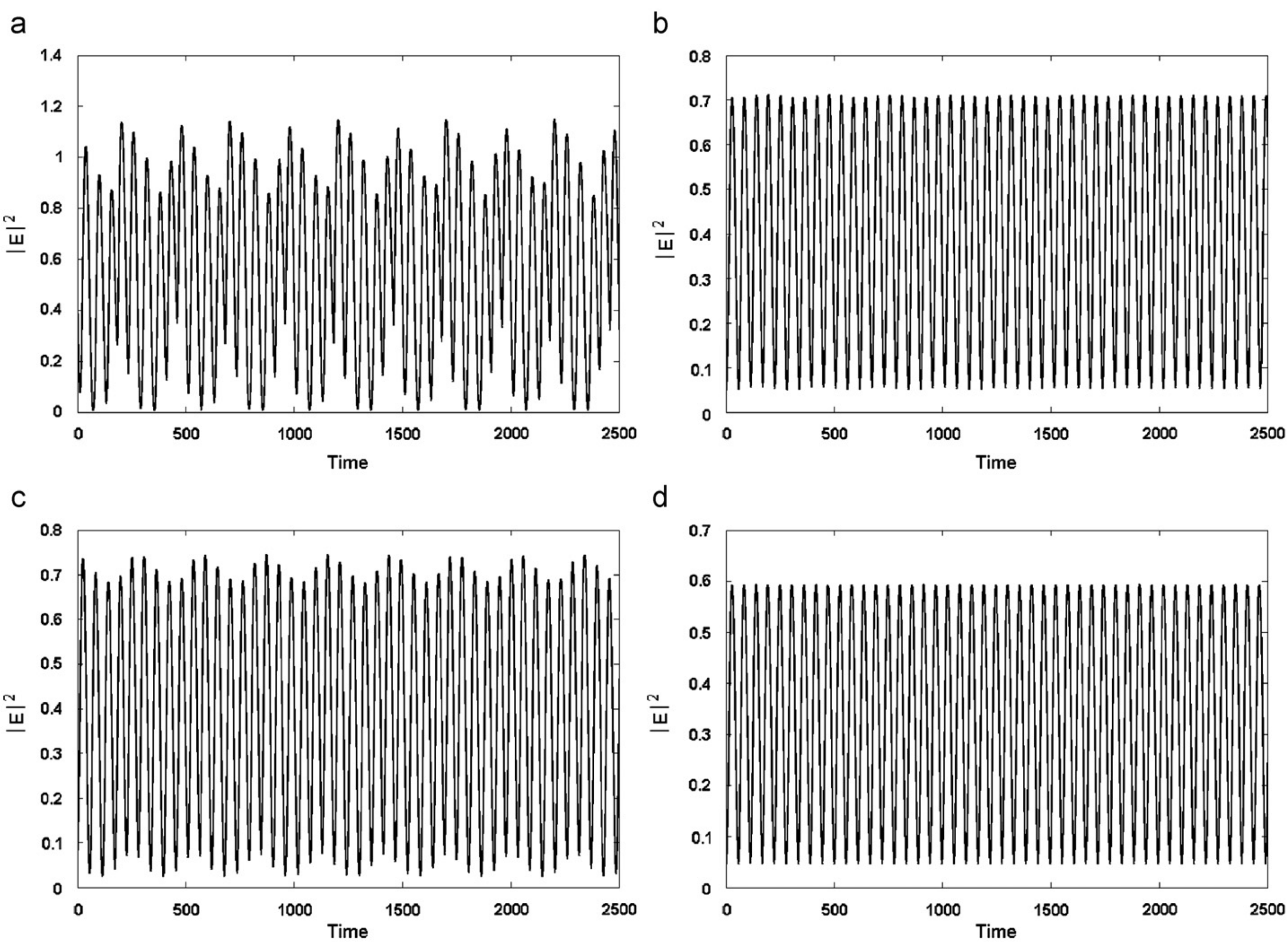

Fig. 5. Time series of $|E^2|$, with $\eta = 0.0455$ and $P = 1.1$ after applying control method with (a) $\varepsilon = 0.02$ and $C_p = -\pi$, (b) $\varepsilon = 0.03$ and $C_p = -\pi$, (c) $\varepsilon = 0.03$ and $C_p = \pi$, (d) $\varepsilon = 0.04$ and $C_p = \pi$.

applying the control method. At $-3.14 < C_p < -1.85$, the dynamics of the laser output is chaotic. As the phase is increased, the chaotic dynamics undergoes QP leading to periodic (P1) area. Finally, the dynamics of the $|E^2|$ undergoes chaos at $1.88 < C_p < 3.14$. In this case, because of increasing the pump current to $P=1.1$, the value of $\varepsilon=0.01$ does not have a suitable effect in reducing the instability. Therefore, $\varepsilon=0.02$, 0.03 and 0.04 are chosen for ε. In comparison with $\varepsilon=0.02$, Fig. 4a, applying the control method leads to removing the chaotic domain at $-3.14 < C_p < -1.85$, and instead, the QP and P1 dynamics appear (Fig. 4b). At the right hand side of this figure ($C_p > 1.88$), the chaotic area is reduced to a narrow column ($2.19 < C_p < 2.54$) and a new QP domain is appeared at $2.608 < C_p < 3.14$. With $\varepsilon=0.03$, as it can be seen in Fig. 4c, the range of P1 is increased to $-3.14 < C_p < -1.85$ and the QP dynamics is removed from this area. In this case, the chaotic zone appeared in Fig. 4a ($1.88 < C_p < 3.14$) is also removed as the area of QP increased. In Fig. 4d, the dynamics of the $|E^2|$ is shown as a function of Cp for $\varepsilon=0.04$. As it can be noticed from this figure, the number of QP oscillation modes is reduced, and at $2.859 < C_p < 3.14$ the QP dynamics is completely converted to P1. We have also investigated the time series of the $|E^2|$ for each value of the ε, and different values of the feedback phase. The time series of the $|E^2|$, with $C_p = -\pi$, indicates a QP behavior for $\varepsilon=0.02$ (Fig. 5a), and a P1 behavior for $\varepsilon=0.03$(Fig. 5b). With positive value of feedback phase, $C_p = \pi$, the time series of the $|E^2|$ shows a QP dynamics for $\varepsilon=0.03$ (Fig. 5c), and a P1 dynamics for $\varepsilon=0.04$ (Fig. 5d). As a result, for negative values of feedback phases, the dynamics of the $|E^2|$ is stable (P1) with $\varepsilon=0.03$. Whereas, for positive values of feedback phase, the dynamics of the $|E^2|$ is P1 with $\varepsilon=0.04$.

4. Feedback strengths as a control parameter

In order to study the role of the control method and ε on the output dynamics of the SL further, we analyzed the dynamics of the $|E^2|$ as a function of η, as a control parameter. For low feedback strengths, it is expected that the output of laser contains chaotic, QP and periodic (P1 and P2) behaviors. Also, for high η, it is expected that the output of the laser contains regular pulse package (RPP), P1 and QP behaviors. Therefore, for controlling the behavior of the laser output by the control technique, we selected the range of the feedback strength to be $0 < \eta < 0.06$, in which the laser displays a chaotic dynamics [27]. After applying the proposed technique, the results is compared with Fig. 6a, to show the dynamic behavior of the $|E^2|$, before the applied technique. Operation of the laser for $0.0104 < \eta < 0.0138$, $0.0318 < \eta < 0.0356$ and $0.0486 < \eta < 0.0534$ is chaotic. These intervals are denoted by zone 1, zone 2 and zone 3 in Fig. 6a, respectively. In figs. 6b–d, the dynamics of the $|E^2|$ is represented as a function of C_p for $\varepsilon=0.01$, 0.02 and 0.03, respectively. For $\varepsilon=0.01$ (Fig. 6b), the chaotic dynamics is repeated at zone 1. At zone 2 the chaotic behavior is converted to P1, which undergoes

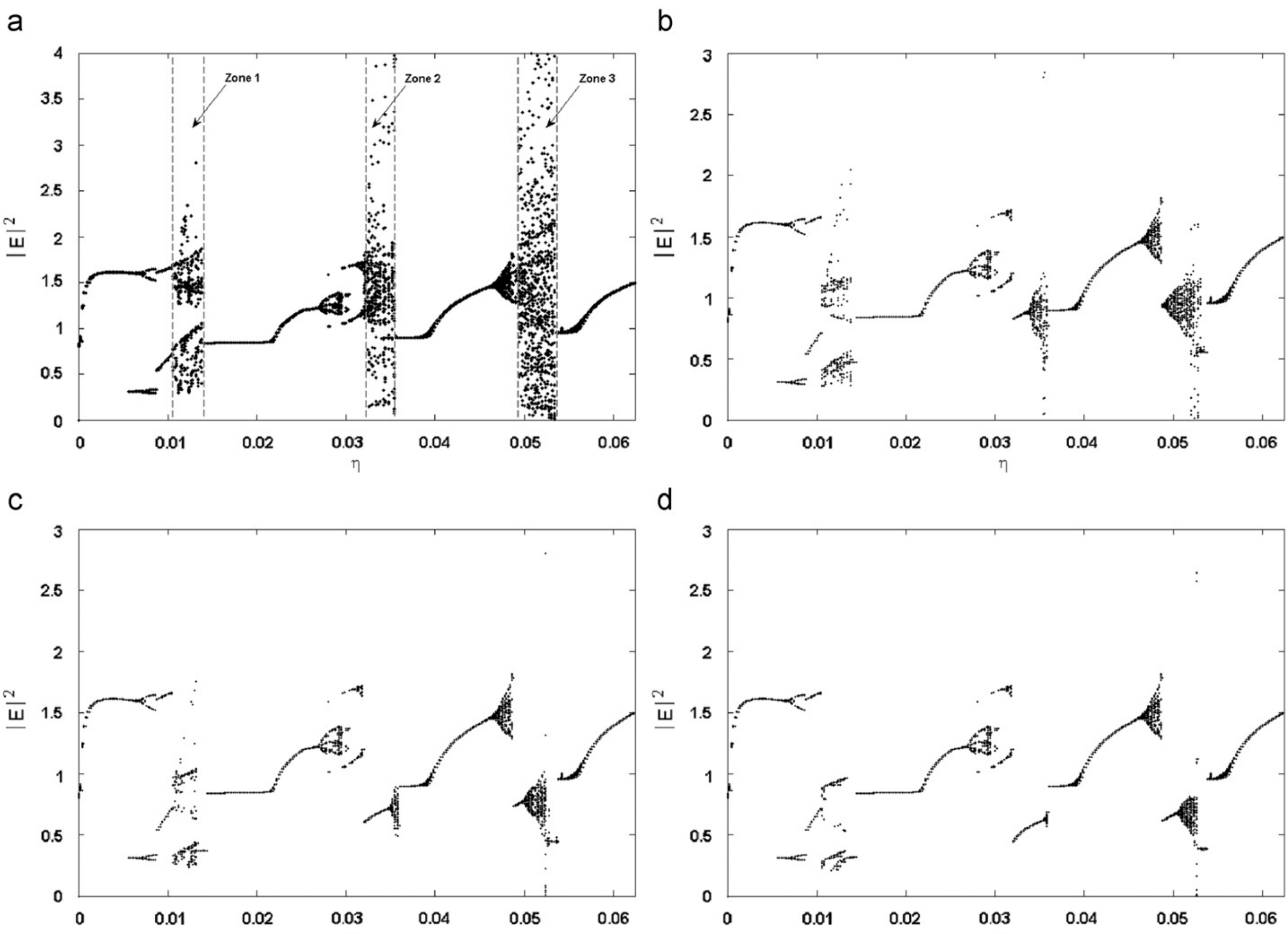

Fig. 6. Bifurcation of $|E^2|$versus η with $C_p=-1.373$ and $P=0.8$, before applying control method (a), after applying control method with $\varepsilon=0.01$ (b), $\varepsilon=0.02$ (c), and $\varepsilon=0.03$ (d).

a QP leading to a small region of chaos ($0.0354 < \eta < 0.0356$). Whereas an extensive area of the zone 3 is converted to QP, a small region $0.052 < \eta < 0.534$ of chaotic dynamics is remained. For $\varepsilon = 0.02$ (Fig. 6c), the chaotic dynamics of zone 1 is nearly repeated, and the QP dynamics appeared in zone 2 is limited to a narrow interval ($0.035 < \eta < 0.0356$), and instead, a P1 dynamics grows. Furthermore, at a branch of zone 3 ($0.0488 < \eta < 0.0496$) the QP dynamics converts to P1. The results for $\varepsilon = 0.02$ are shown in Fig. 6c. Finally, with $\varepsilon = 0.03$ in the control method, a branch of zone1 ($0.0104 < \eta < 0.0138$) in Fig. 6a converts to periodic (P1, P2, P3 and P4), zone 2 completely converts to P1, the QP domain at zone 3 is reduced, and P1 is increased to $0.0488 < \eta < 0.0504$ interval. The results of this regime are indicated in Fig. 6d.

We also investigated max $|E^2|$ as a function of P for before and after applying the control method. Fig. 7 shows variation of max$|E^2|$ versus P. This figure is plotted for the parameters of $C_P = -1$ and $\eta = 0.036$. Before applying the control method, max$|E^2|$ increased gradually as P increases for $P < 0.4$, whereas for $P > 0.4$ the increment of max$|E^2|$ is almost tangential. This shows that the output of the laser is stable for $P < 0.4$ and instability starts for $P > 0.4$. it can be seen from Fig. 7, the stability of max$|E^2|$ has increased for $P > 0.8$, after applying the control method. However, the general variation of the graph with respect to prior applying the control method has not changed.

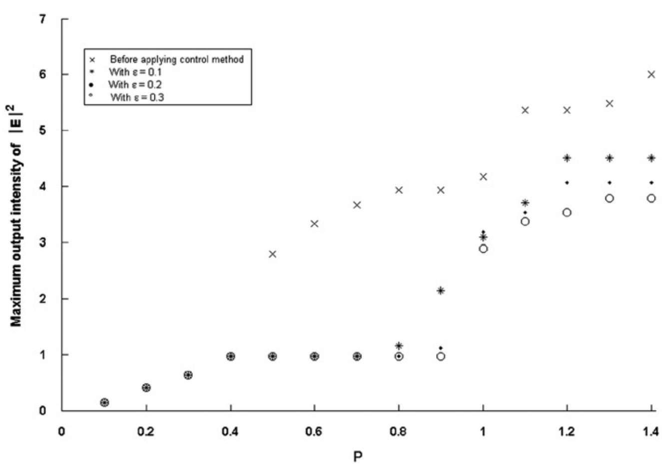

Fig. 7. Diagram of maximum output intensity $|E^2|$ versus P with $C_p = -1$ and $\eta = 0.036$.

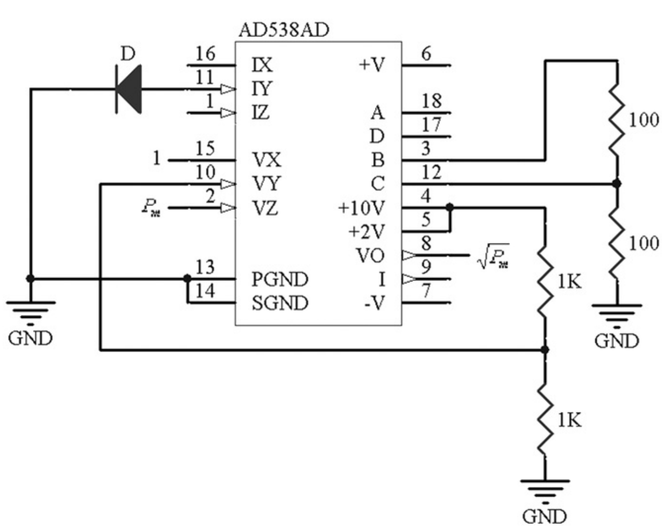

Fig. 8. Generator of $V_0 = \sqrt{P}$, with input value of $V_Z = P_m$, in which $V_0 = V_Y\sqrt{V_Z/V_X}$ and $V_X = V_Y = 1$.

5. Conclusion

We studied the chaos controlling and stability of the laser output by a variable parameter denoted by ε in chaotic map (Eq. (3)), which controls the laser pump current by an electronical circuit and an optical setup. It has been shown that the laser output in terms of the feedback phase and strength can be chaotic [27]. The control method introduced here has been designed in such a way that can be applied to the laser at the intervals of the C_p and η, in which the laser output is chaotic. The bifurcation curves and time series of the laser output have shown that the chaotic dynamical behavior of the laser can be totally converted to a stable state using the control method. It should be noted that the ε has an important role in the implementation of the controlling of the laser output dynamically. The selection of initial value for pump current (P) has a direct influence on the chaotic

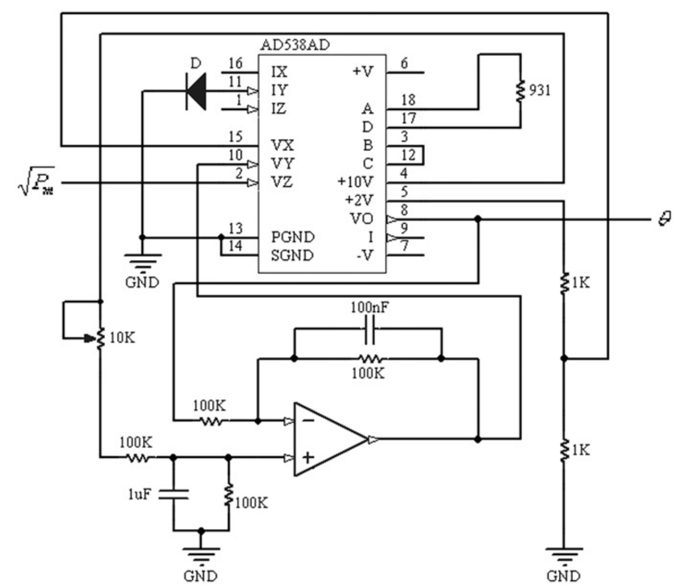

Fig. 9. Generator of $V_0 = \arctan\sqrt{P}$, with input value of $V_Z = \sqrt{P_m}$, in which $V_0 = \arctan(V_Z/V_X)$ and $V_X = 1$.

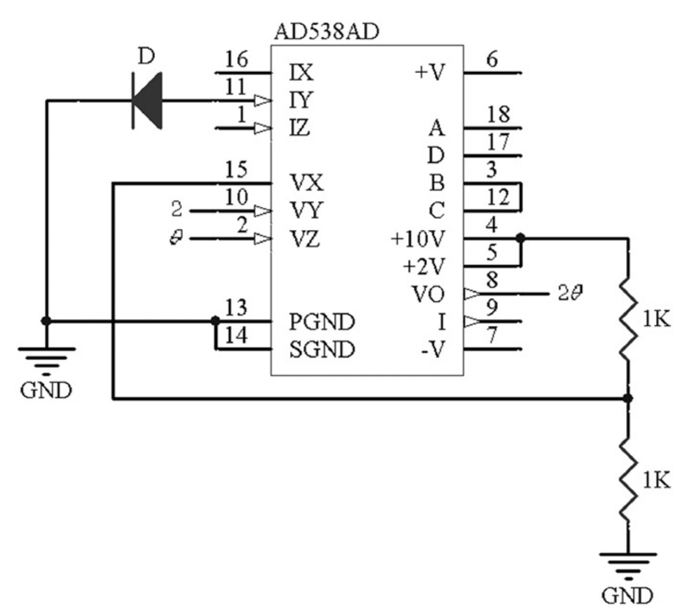

Fig. 10. Generator of $V_0 = 2\theta$, with input value of $V_Z = \theta$, in which $\theta = \arctan\sqrt{P_m}$, $V_0 = V_Y(V_Z/V_X)$, $V_Y = 2$ and $V_X = 1$.

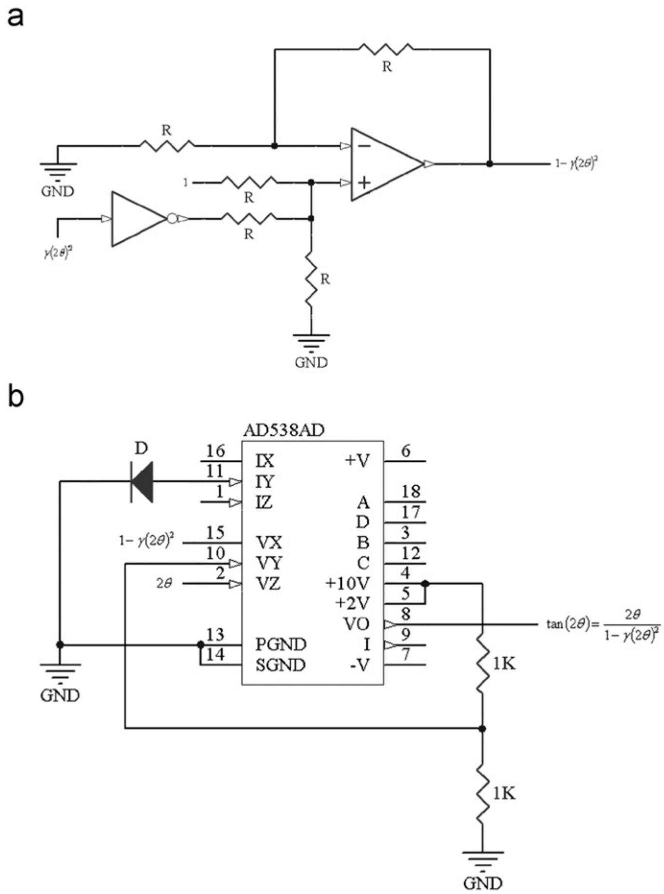

Fig. 11. Generator of $\tan 2\theta$ function, with $\theta=\arctan\sqrt{P_m}$, (a) generator of $1-\gamma(2\theta)^2$, with $\gamma=0.38$ and different input values of $\gamma(2\theta)^2$, (b). The generator of $\tan 2\theta$, $\tan 2\theta=(2\theta)/(1-\gamma(2\theta)^2)$, with $V_0=V_YV_Z/V_X$, $V_Y=1$, $V_Z=2\theta$, and $V_X=1-\gamma(2\theta)^2$.

range of the laser dynamics. The chaotic range of the laser output increases as the P (as an initial value of pump current) increases. We have investigated the influence of the ε in the increase of the chaotic range in different initial pump currents. Comparing the dynamical behavior of the laser output before and after the control method shows that we may have stable outputs for the laser with different initial values of the pump currents. We have emphasized that the introduced control method in this paper has not designed only for an especial chaotic map, but it can be used for different chaotic maps. The most important advantage of this method is manual control of its dynamical parameter.

Appendix A. Detail of control circuit

The CMGC consist of square root (Fig. 8), arctan (Fig. 9), multiplier (Fig. 10), substrater (Fig. 11a), and divider (Fig. 11b) operators.

References

[1] Donati S, Mirasso CR. IEEE Journal of Quantum Electronics 2002;38:1138–40.
[2] Buldu JM, Garcia-Ojalvo J, Torrent MC. IEEE Journal of Quantum Electronics 2005;41:164–70.
[3] Jia-Gui W, Guang-Qiong X, Liang-Ping C, Zheng-Mao W. Optics Communications 2009;282:3153–6.
[4] Banerjee S, Rondoni L, Mukhopadhyay S, Misra AP. Optics Communications 2011;284:2278–91.
[5] Banerjee S, Rondoni L, Mukhopadhyay S. Optics Communications 2011;284:4623–34.
[6] Tang S, Liu JM. IEEE Journal of Quantum Electronics 2001;37:329–36.
[7] Torcini A, Barland S, Giacomelli G, Marin F. Physical Review A 2006;74: 063801–13.
[8] Green K. Physical Review E 2009;79:036210–2.
[9] Heil T, Fischer I, Elsäßer W, Krauskopf B, Green K, Gavrielides A. Physical Review E 2003;67. 0662141-11.
[10] Heil T, Fischer I, Elsäßer W. Physical Review Lett. 2001;87:243901–4.
[11] Ott E. Physical Review Letters 1990;64:1196–9.
[12] Uchida A. Physical Review E 1998;58:7249–55.
[13] Kuomo KM, Oppenheim AV. Physical Review Letters 1993;71:65–8.
[14] Lynch S, Steele AL. Chaos. Solitons & Fractals 2000;11:721–8.
[15] Ciofini M, Labate A, Meucci R, Arecchi FT. European Physical Journal D 1999;7:5–9.
[16] Rajesh S, Nandakumaran VM. Physica D 2006;213:113–20.
[17] Dykstra R, Tang DY, Heckenberg NR. Physical Review E 1998;57:6596–8.
[18] Martinez-Zerega BE, Pisarchik AN, Tsimring LS. Physical Letters A 2003;318:102–11.
[19] Pisarchik AN, Goswami BK. Physical Review Letters 2000;84:1423–6.
[20] Pisarchik AN. Physical Letterss A 2003;318:65–70.
[21] Martinez-Zerega BE, Pisarchik AN. Physical Letters A 2005;340:212–9.
[22] Gauthier DJ, Sukow DW, Concannon HM, Socolar ES. Physical Review E 1994;50:2343–6.
[23] Lang R, Kobayashi K. IEEE Journal of Quantum Electronics 1980;16:347–55.
[24] Verduyn Lunel SM, Krauskopf B. In: Krauskopf B, Lenstra D, editors. Fundamental Issues of Nonlinear Laser Dynamics, AIP Conference Proceedings, vol. 548. Melville, NY: American Institute of Physics; 2000. p. 66.
[25] Erneux T. Physical Review E 2004;69:036210–5.
[26] Kuznetsov Yu. Elements of Applied Bifurcation Theory. Berlin: Springer; 2005.
[27] Jafari A, Sedghi H, Mabhouti Kh, Behnia S. Optics Communications 2011;284:3018–29.
[28] Jafarizadeh MA, Behnia S. Physica D 2001;159:1–21.
[29] Behnia S, Akhshani A. Chaos Solitons & Fractals 2009;42:2105–14.