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ORIGINAL SCIENTIFIC RESEARCH

Effect of quantum correction of electron–ion collision frequency on ion temperature and expansion energy in laser-driven fusion

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Abstract: The nonlinear heat conduction equation can be solved by quantum modified heat transfer coefficient in deuterium–tritium (DT) plasma driven by laser above the critical electron temperature. As a result, the electron profile temperature and the heat flux solutions are improved. In this paper, we have studied the subsequent modification of ion temperature due to ion heating by energy transfer between electrons and ions through electron–ion collisions. The differential equation between T_e and T_i, expansion energy of plasma as well as the threshold energy of the laser beam for plasma ignition are renewed by considering quantum modification in electron–ion collision frequency. The achieved results improve the values of the ion temperature and the threshold energy for plasma ignition compared with the values obtained from the calculations using the classical electron–ion collision frequency.

Keywords: Laser-driven plasma; Threshold energy; Plasma ignition; Collision frequency

1. Introduction

When a very intense laser beam impinges on a plate, hot electrons are instantly generated on the surface of the sample. At this moment, the thermal waves and the heat flux of electrons transfer energy from the laser beam into the plate's material. Energy and momentum transferred from the laser light into the solid material have been investigated in Ref. [1]. In the past two decades, high-density plasma and high-temperature plasma have been produced by the advent of amplified chirped-pulse lasers [2]. The study of such plasma systems is required to understand the interaction between intense ultrashort laser pulses and the plasma before the significant hydrodynamic expansion of plasma takes place [3]. During very short periods of the order 1 ps, the heat transport is dominant [4], and for other periods much longer than the equipartition time, one has to take the hydrodynamic motion energy in theoretical modeling into account. However, since the high-temperature gradients are created in the location of laser beam absorption, thermal waves and heat flux of electrons are expected to run ahead of the radiation. In addition, for periods which are longer than the laser duration time, hydrodynamic motions of electrons just begin in the plasma.

Previously, gaseous targets and metallic surfaces have been extensively used in the study of matter interaction with powerful Q-switched laser beams. Furthermore, the ionization and heating of solid materials by means of a laser pulse have been investigated widely in Refs. [5–7]. Zeldovich et al. [8] have presented an analytic solution for nonlinear heat transport dynamics with imposing merely some mathematical boundary conditions which cannot be performed in a real experiment. Moreover, Mayer et al. [9] have obtained another solution for the heat transport equation with suitable initial boundary conditions having physical concepts.

Moreover, some various mechanisms have been proposed to explain laser-driven ion acceleration, including target normal sheath acceleration (TNSA) enhanced by the laser parameter optimization [10, 11], Coulomb explosion (CE) [12], skin layer ponderomotive acceleration (SLPA) [13], and radiation pressure acceleration (RPA). In SLPA regime, the ponderomotive force near critical plasma surfaces (induced by inhomogeneity of the laser field) efficiently accelerates the electrons and ions [14]. In SLPA mechanism, the electrostatic field, caused by charge

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separations due to the ponderomotive force, accelerates the ions. Furthermore, proton acceleration by overdense plasma target interacting with shaped laser pulses has been investigated by Kumar et al. [15]. In this research, pre-plasma has been introduced as a linearly increasing plasma density ramp in front of the overdense plasma target and the shaping of laser pulse is introduced by considering different durations of leading and trailing pulse edge. Besides, oscillating two-stream instability (OTSI) of a high-amplitude laser or a plasma wave in plasmas with strongly coupled ions has been studied by Sharma et al. [16] as another mechanism for ion acceleration. For short laser pulses, when the thermal absorption is dominant, energy equipartition between electrons and ions plays an important role in ion heating.

Bobin [1] has also formulated a differential equation, besides the ion–electron relaxation, in which the ion temperature could be calculated via the electron temperature considered in the deflagration structure. The electron and ion temperature profiles as well as the threshold energy of the laser beams for the plasma ignition have been computed at different times by Chu [17]. The first quantum modification of electron–ion collision frequency (EICF) in a fully ionized plasma has been introduced by Bethe [18] and observed in the Stellarator when Grieger et al. [19] measured the diffusion of plasma across a magnetic field. The thermal conductivity coefficient related to the modified collision frequency of electrons has been confirmed experimentally by Razumova [20]. In addition, the effect of quantum correction on nonlinear heat waves of electrons has been investigated by Nafari et al. [21] in a laser-driven (inertial) fusion. In their research, the electron temperature profiles have been calculated in detail with quantum consideration of EICF.

In the present research, we have first reconsidered the ion–electron relaxation equation describing the relation between T_e and T_i by conserving the mass, momentum, and energy of the plasma system. This equation consists of a thermal conduction term and another thermodynamically expansion term of plasma according to the well-known formula presented in Ref. [1]. Quantum correction effect in earlier research [21] leads to a term proportional to $T^{3/2}$ for the thermal conduction coefficient. An equation obtained by quantum mechanical heat transfer is combined with the ion heating equation, presented in [22], to obtain a modified differential equation connecting T_e and T_i. Our numerical results reveal that the ion temperature is about two times greater than the amounts obtained in [17]. The results of Ref. [21] show that at 0.08 ns, the electron temperature increases about 10 times greater than the classical values. Moreover, the laser threshold energy has been investigated in Refs. [21, 23], but they did not take the corrected expansion energy of plasma in energy balance equation into account due to the loss and the reabsorption of thermonuclear energy. Here, the quantum correction of energy loss of expanding plasma was also extended by considering the thermal conductivity of heat transfer in a DT plasma. In Ref. [17], the dependence of expansion energy on the electron temperature has been derived as $T^{1/2}$. We have also shown that this dependence has to be identified as $T^{+1/2}$ (see "Appendix"). Furthermore, we have demonstrated that the threshold energy of a laser beam for the plasma ignition is decreased due to the improved dependence of expansion energy of DT plasma.

This paper is organized as follows. In Sect. 2, we have reviewed the EICF and the quantum considerations to calculate the thermal conductivity function. In Sect. 3, we have introduced the modified ion heating temperature and the expansion energy of plasma. In Sect. 4, we present the improved ignition condition in DT plasma due to quantum consideration on EICF for laser-driven threshold energy. Section 5 is devoted to the description of the mathematical methods and the numerical results. Finally, Sect. 6 provides a summary of results and conclusions.

2. Preliminaries

Here, the methods applied for calculating the heating of electrons via a heat flux produced by absorbing the laser light in a solid DT target are briefly presented. We have first focused on quantum EICF and thermal conductivity of electrons from classical and quantum–mechanical points of views. Then, we have briefly reviewed the effect of quantum EICF on the solutions of nonlinear thermal conduction equation in the fully ionized plasma.

2.1. Quantum EICF and thermal conductivity

In the fully ionized plasma, Bethe [18] has formulated the EICF by introducing the electron–ion impact parameter r_o for a collision limited to a 90° hyperbolic motion. Bethe concluded that with an impact parameter of the order or less than de Broglie wavelength $\lambda_{dB} = h/(2m_eE)^{1/2}$, the EIC results in a 90° scattering. Therefore, in the fully ionized plasma, the quantum impact parameter $h/2(3m_ek_bT)^{1/2}$ is replaced with the classical one $Ze^2/3k_bT$ corresponding to a 90° hyperbolic motion, where h, m_e, E, e, T, and k_b are Planck's constant, mass of electron, electron energy, charge of electron, electron temperature, and Boltzmann's constant, respectively.

As it was pointed out by Marshak for the first time [24], the classical and quantum impact parameters are equal to a special electron temperature:

$$T^* = \left(\frac{4}{3k_b}\right) m_{oe} c^2 Z^2 \alpha^2 \tag{1}$$

where $\alpha = e^2/\hbar c$ is the fine structure constant and c is the speed of light. For the case of a DT plasma with $Z = 1$, T^* is determined as 35.9 eV. In Refs. [25–28], a general formula for EICF has been derived as:

$$\upsilon_{ie} = 0.785 \frac{n_e a_B^2}{Z^3} \left(\sqrt{\frac{3k_b T}{m_{0e}}}\right) \left[\left(1 + \frac{4T}{T^*}\right)^{1/2} - 1\right]^{-2} \tag{2}$$

where n_e is the electron density and a_B is the Bohr radius. Applying Taylor expansion and using the general EICF calculated by Hora [27] in a 90° wave diffraction model, both of the classical and quantum EICF with the critical temperature T^* can be written as follows:

$$\upsilon_{\mathrm{ie}} = \begin{cases} \upsilon_{\mathrm{cl}} & \text{if } T < T^* \\ \upsilon_{\mathrm{cl}} \frac{T}{T^*} & \text{if } T > T^* \end{cases} \tag{3}$$

In Eq. (3), the calculated EICF, under some approximations and $\upsilon_{\mathrm{cl}} = \pi n_e Z e^4 / 3^{3/2} m_e^{1/2} (k_b T)^{3/2}$, is in a very good agreement with the classical one derived by Spitzer and Harm [22] for a small angle of scattering that leads to:

$$\upsilon_{\mathrm{cl}} = \frac{\pi^{3/2} n_e Z e^4 \ln \Lambda}{2^{5/2} m_e^{1/2} (k_b T)^{3/2}} \tag{4}$$

where $\ln \Lambda$ is the Spitzer's Coulomb logarithm given as:

$$\Lambda = \frac{\lambda_d}{r_o(90^\circ)} = \left(\frac{3k_b^3 T^3}{2\pi Z^2 e^6 n_e}\right). \tag{5}$$

Here, $\lambda_d = (k_b T / 4\pi n_e e^2)^{1/2}$ is the Debye length. The Spitzer's Coulomb logarithm has values between 5 and 20 for various values of T and n_e. The necessity of the quantum modified EICF in high electron temperatures has been initially notified by Grieger et al. [19] in a Stellarator when they were investigating the diffusion of a tokamak plasma across a magnetic field. They pointed out that the value of experimental diffusion velocity is 20 times greater than the classical value for Deuterium plasma at a temperature about 800 eV. In addition, they announced this experiment with the empirical ratio of $T/T^* = 22$ which provides a convincing proof of quantum mechanical feature of EIC. The other quantity that has to be considered quantum mechanically is the thermal conductivity K_e of electrons calculated in Ref. [19] as follows:

$$K_e = K_{\mathrm{cl}} \left(\frac{T^*}{2T}\right) \left[\left(1 + \frac{4T}{T^*}\right)^{1/2} - 1\right]^2. \tag{6}$$

After Taylor's expansion in terms of T, the K_e can be approximated as follows:

$$K_e = \begin{cases} (1.89)\left(\frac{2}{\pi}\right)^{\frac{3}{2}} \frac{k_b^{\frac{7}{2}} T^{\frac{5}{2}}}{e^4 m_e^{\frac{1}{2}} \ln \Lambda} = K_{\mathrm{cl}} & \text{if } T < T^* \\ (20)\left(\frac{2}{\pi}\right)^{\frac{3}{2}} \frac{4Z^2 m_e^{\frac{1}{2}} k_b^{\frac{5}{2}} T^{\frac{3}{2}}}{3\hbar^2 \ln \Lambda} = K_{\mathrm{cl}} \frac{T^*}{T} & \text{if } T > T^* \end{cases} \tag{7}$$

The decrease in quantum thermal conductivity of electrons, in comparison with the classical one, has been explored experimentally by Razumova [20]. They found that the thermal conductivity of electrons in a high-beta tokomak parallel to the magnetic field [22] is up to 20 times less than the classical one.

2.2. Solution of nonlinear heat conduction equation for hot electrons in one dimension

The energy of laser light absorbed by electrons generates a temperature gradient and subsequently heat flux on the surface of solid target. As a result of this phenomenon, heat transportation occurs between the cold electrons and the other electrons heated by the laser beam. In the case of semi-infinite target with a one-dimensional scheme, the heat flux φ_H in the plasma is given by $\varphi_H = -\chi(\partial T_e/\partial x)$. The energy balance equation reads as:

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n k_b T_e\right) + \frac{\partial \varphi_H}{\partial x} = 0. \tag{8}$$

Introducing $T' = \left[\frac{3}{2}\right] n k_b T_e$, $\chi' = \chi\left(\frac{3}{2} n k_b\right)^{-1}$, we have the heat diffusion equation as follows:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\chi \frac{\partial T}{\partial x}\right). \tag{9}$$

In general, χ is not a constant coefficient and without losing generality we can define it as a function of plasma temperature in the form of $\chi = aT^n$. Here, a is a constant that has the dimension of (K^n cm^2 s^{-1}) in cgs units. Also, n is a real number that its value specifies the kind of physical situation of the problem. For example, the values of $n = 4$, 5 are related to radiation flux and the values of $n = 5/2$, $n = 3/2$ stand for electron heat transfer problem with classical and modified heat conductivity, respectively. However, the problem of heat conduction with both classical and modified heat conduction coefficients is classified by substituting $n = 5/2$ and 3/2 in Eq. (9), respectively, as follows:

$$\frac{\partial T}{\partial t} = a\frac{\partial}{\partial x}\left(T^{5/2}\frac{\partial T}{\partial x}\right), \quad \frac{\partial T}{\partial t} = a\frac{\partial}{\partial x}\left(T^{3/2}\frac{\partial T}{\partial x}\right). \tag{10}$$

The solutions of Eq. (10) depend on the manners of laser light deposit energy in the solid target. The suggested manners construct the forms of boundary conditions that we impose on Eq. (10) before attempting to solve the heat transfer problem. There are two kinds of different boundary conditions [8, 9] that can be imposed on Eq. (10) to solve the electron heat conduction problem. We can derive Chu's solutions again by choosing the boundary condition as the Dirac delta function of x [8]. The solution of Eq. (10) with Chu's boundary condition assumption can be written as:

$$T = \left(\frac{Q^2}{at}\right)^{\frac{1}{n+2}} f(\xi) \tag{11}$$

where the dimensionless function $f(\xi)$ is defined as follows:

$$f(\xi) = \left[\frac{n\xi_o^2}{2(n+2)}\left(1 - \frac{\xi^2}{\xi_o^2}\right)\right]^{\frac{1}{n}} \tag{12}$$

where ξ is a variable which can be defined as:

$$\xi = \frac{x}{(Q^n at)^{\frac{1}{(n+2)}}}. \tag{13}$$

Also, ξ_o is a constant parameter which can be evaluated by energy conservation considerations. At initial points of time, when ions are cold, and assuming that the total deposit energy of laser beam is absorbed by the electrons, Q can be derived by considering energy conservation as follows:

$$Q = \int_{-\infty}^{+\infty} T_e \mathrm{d}x \tag{14}$$

with

$$Q = \frac{E_{in}}{\left(\frac{3}{2}\frac{\rho k_b}{m_i}\right)} \tag{15}$$

where ρ represents the plasma density. When scales are larger than the laser pulse duration, Eq. (14) must be replaced by the following equation:

$$Q = \int_{-\infty}^{+\infty} (T_e + T_i)\mathrm{d}x. \tag{16}$$

The solution obtained as Eq. (11) is singular at $t = 0$, which means that the value of temperature as a physical quantity is infinite at $t = 0$ as mentioned in Ref. [21]. The boundary condition described in Ref. [8] seems to belong to a pure mathematical problem rather than a physical heat transfer problem. Mayer et al. [9] solved this difficulty by modifying the boundary conditions of Eq. (10). They assumed that the laser beam deposits energy in length L_o of solid target. They also obtained a finite temperature for electrons at the initial time $t = 0$ and $x = 0$. The solution of Eq. (10), obtained by Mayer's assumptions, can be derived as follows:

$$T(x,t) = \frac{T_o}{Z(t)}\left[1 - \frac{x^2}{x_f^2(t)}\right]^{\frac{1}{n}} \tag{17}$$

where the front of the heat wave moves according to

$$x_f(t) = \left(\frac{2}{n}\right)^{\frac{1}{2}} L_o Z(t) \tag{18}$$

with

$$Z(t) = \left[1 + (2+n)\left(\frac{aT_o^n}{L_o^2}\right)t\right]^{\frac{1}{(n+2)}} \tag{19}$$

where T_o is the initial temperature of electron thermal wave calculated according to the energy conservation considerations at time $t = 0$ as follows:

$$T_o = \frac{\left(\frac{n}{2}\right)^{1/2}}{L_o} Q \left[\frac{\Gamma\left(\frac{1}{n}+\frac{3}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{n}+1\right)}\right]. \tag{20}$$

In all the formulas, L_o is an initial length scale which is the same as wavelength of the driver laser. In summary, the numerical results of electron temperature for a DT plasma determined by Eqs. (11) and (17) can be presented by Figs. 1 and 2 as follows:

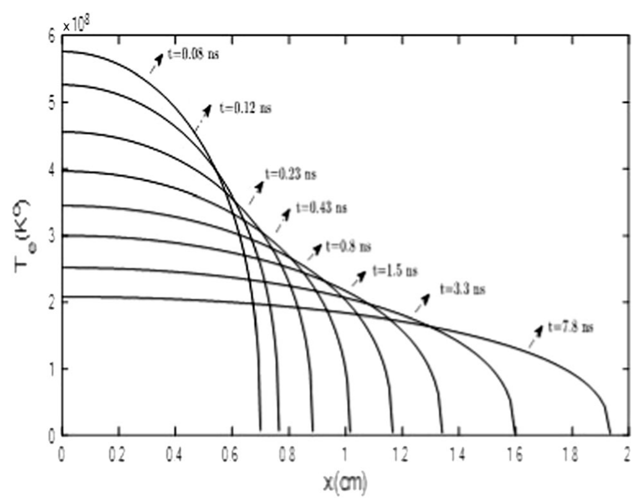

Fig. 1 Computed electron temperature profiles at different times with classical thermal conductivity as confirmed by Ref. [18]

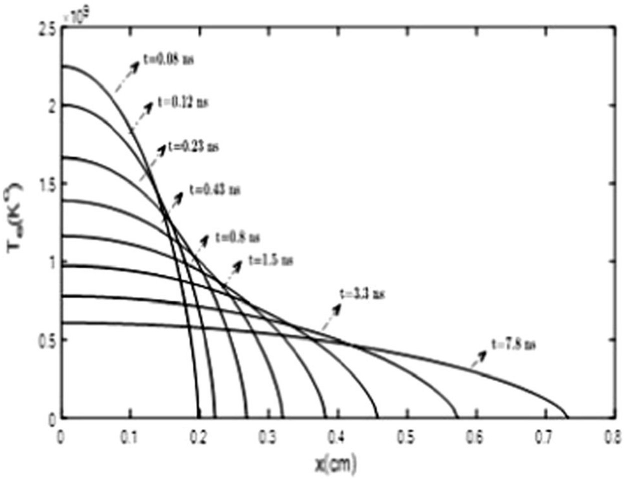

Fig. 2 Computed electron temperature profiles at different times with effect of quantum EICF modification on thermal conductivity

3. Electron–ion thermal exchange energy by collisions

The electrons and ions in plasma are not in thermal energy equilibrium, so there is a net exchange of thermal energy through Coulomb collisions between electrons and ions [31]. In the area of interaction structure on a solid DT plate where the laser light interacts only with the target electrons, it has been assumed that the temperature of electrons is not the same as of ions. The interacted electrons acquire energy and momentum from the laser light, and since *e–e* collision frequency is very high, the velocity distribution becomes a Maxwellian function within a very short and negligible timespan even if the interaction mechanism is a Compton effect which prevails over the higher fluxes [32]. Energy equipartition between two species is the only available mechanism from which the ions can be heated by the electrons. Moreover, this is slow and occurs with a characteristic time denoted τ_{eq} which depends on the power of electron temperature as 3/2. We can write the characteristic time [1] as follows:

$$\tau_{eq} = \frac{3m_e m_i k^{\frac{3}{2}}}{8(2\pi)^{\frac{1}{2}} n_e e^4 \ln\Lambda} \frac{k}{m_i} \left(\frac{T_e}{m_e} + \frac{T_i}{m_i}\right)^{\frac{3}{2}}. \quad (21)$$

Here, m_e and m_i are the mass of electron and ion, respectively. For deuterium (D_2), m_i is 3600 times greater than m_e. The ion temperature T_i, according to the previous assumptions, is always smaller than T_e. Therefore, we can neglect it in the second term inside the parentheses of Eq. (21) as follows:

$$\tau_{eq} = \frac{3m_e m_i k_b^{\frac{3}{2}}}{8(2\pi)^{\frac{1}{2}} n_e e^4 ln\Lambda} \left(\frac{T_e}{m_e}\right)^{\frac{3}{2}} = \frac{BT_e^{\frac{3}{2}}}{\rho ln\Lambda} = \frac{bT_e^{\frac{3}{2}}}{\rho} \quad (22)$$

with $b = B/\ln\Lambda$ and $B = 1.6 \times 10^{-21}$ (°$K^{-3/2}$ cm^{-3} s). In the framework of ion–electron collisions, the ion heating equation can be used after Spitzer [26] as follows:

$$\frac{dT_i}{dt} = (T_e - T_i)/\tau_{eq}. \quad (23)$$

For high temperatures of ions doped in proton plasmas, this characteristic time has been improved by Landau–Spitzer (LS) theory. Assuming a simple two-body electron–ion operator, an equilibration rate has been derived from Fokker–Planck equation [33]. In this paper, the data were limited to classical characteristic time because the ions in the studied DT plasma are cold.

As the ions are driven with the local velocity of u across the deflagration structure [1], they move from the intermediate medium to the plasma phase with relation. Since and , Eq. (23) can be reduced to:

$$\frac{BT_e^{\frac{3}{2}}}{\ln\Lambda} \frac{dT_i}{dx} = \frac{\rho^2}{J}(T_e - T_i). \quad (24)$$

In the two-fluid model with constant pressure and using in the state equation of plasma [1], we can obtain the following equation:

$$\rho = \frac{2}{3}\left[\frac{m_i p_1}{k_b(T_e + T_i)}\right]. \quad (25)$$

Substituting (26) in (25), we have:

$$\frac{BT_e^{\frac{3}{2}}}{\ln\Lambda} \frac{dT_i}{dx} = \frac{4p_1^2}{9J} \frac{(T_e - T_i)}{(T_e + T_i)^2}. \quad (26)$$

In the constant pressure approximation regime with sufficient accuracy in Ref. [1], dealing with electron heat conduction and transporting terms of driven plasma, the main energy conservation equation can be obtained as follows:

$$\frac{A}{ln\Lambda} T_e^{\frac{5}{2}} \frac{dT_e}{dx} = \frac{1}{4} J \frac{5\gamma - 1}{\gamma - 1} \frac{k_b}{m_i}(T_e + T_i). \quad (27)$$

In Eq. (27), varies slowly with electron temperature, so it can be assumed as a constant number in the thermal conduction process.

Finally, the differential equation linking T_e to T_i is obtained by dividing two sides of Eqs. (24) to (27) as follows:

$$\frac{dT_i}{dT_e} = \frac{16}{9} \frac{A_{cl}}{BJ^2} \frac{\gamma - 1}{5\gamma - 1} \left(\frac{m_i}{k_b}\right)^3 p_1^2 \frac{(T_e - T_i)}{(T_e + T_i)^2}. \quad (28)$$

By assuming Eqs. (11) or (20) as solutions for T_e, obtained in Sect. 3, and multiplying two sides of Eq. (28) to, we can solve Eq. (28) to obtain ion temperature T_i as a function of x and t, $T_i(x,t)$. In Sect. 5, the modified form of Eq. (28) was used in our calculations which can be derived by choosing the modified heat conduction coefficient in the left-hand side of Eq. (24).

4. Improved ignition condition due to quantum consideration on EICF for laser-driven threshold energy in DT plasma

The fundamental equation of threshold ignition condition, as used in [17], has been obtained by examining the system of thermonuclear reaction equations under the assumption $T_e = T_i = T$ at $x = 0$ as:

$$W_e + W_i = A\rho T^{\frac{1}{2}} + \frac{4}{3}T\frac{\partial u}{\partial x} - \frac{2m_i}{3k\rho}\frac{\partial}{\partial x}\left(k_e\frac{\partial T}{\partial x}\right) \tag{29}$$

where W_e and W_i functions are related to reaction rate function, W. The functions, W_e and W_i, are defined in terms of W, and the fraction of burned material, Y, as follows:

$$\begin{aligned} W &= \frac{1}{2}n(1-Y)^2\langle\sigma v\rangle, \\ W_i &= \left(\frac{2W}{3k_b}\right)[(1-f)E_\alpha + E_n\delta_l], \\ W_e &= \left(\frac{2W}{3k_b}\right)fE_\alpha, \end{aligned} \tag{30}$$

where E_α is the energy of alpha particles and E_n is the energy of neutrons. Under the ignition condition, Eq. (29) indicates the fact that the energy gain due to the reabsorption of thermonuclear energy should be just equal to the energy loss in the system of DT plasma. It is obvious that the total lost energies included on the left-hand side of Eq. (29) are the Bremsstrahlung radiation energy, the expanding energy of electrons in plasma system, and the heat conduction energy of electrons, respectively. The simplified form of Eq. (29) can be obtained by substituting the solution of electron thermal conduction equation in Eq. (29) as follows [17]:

$$W_e + W_i = A\rho T^{\frac{1}{2}} + \frac{9}{8}\left(\frac{k_b}{m_i}\right)\left(\frac{1}{a_{cl}T^{1/2}}\right) + \frac{2}{9}\frac{T}{t}. \tag{31}$$

With quantum considerations in EICF, the Mayer solution changes the expanding energy which can be derived from in Eq. (29). By substituting the derived expanding energy term from Eqs. (58) in (29), we have obtained the improved ignition condition equation, due to the quantum consideration in EICF, as follows:

$$W_e + W_i = A\rho T^{\frac{1}{2}} + \frac{12}{7}\left(\frac{k_b}{m_i}\right)\left(\frac{T^{1/2}}{a_{qu}}\right) + \frac{2}{9}\frac{T}{t}. \tag{32}$$

Comparing (31) with (32), we have found that the temperature dependence of expanding energy changes from $T^{-1/2}$ to $T^{+1/2}$ as a result of quantum improvement in expanding energy loss of the electrons. In the next section, it was observed that how this correction changes the numerical calculations.

5. Mathematical methods and numerical results

Equation (24) is accurately reconsidered as follows:

$$\frac{A_{\mathrm{cl}}}{\ln\Lambda}T_e^{\frac{5}{2}}\frac{\mathrm{d}T_e}{\mathrm{d}x} = \frac{1}{4}J\frac{5\gamma-1}{\gamma-1}\frac{k_b}{m_i}(T_e+T_i). \tag{33}$$

We have found that the quantum EICF also changes the thermal conduction energy term in Eq. (27). Moreover, Eq. (27) still satisfies the energy conservation as mentioned in Sect. 4. In Eq. (33), is the classical thermal conduction coefficient defined in Eq. (7). Using Eqs. (9) and (7), we have obtained the modified form of Eq. (27) by substituting for according to the quantum improvement in EICF as follows:

$$\frac{A_{qu}}{ln\Lambda}T_e^{\frac{3}{2}}\frac{\mathrm{d}T_e}{\mathrm{d}x} = \frac{1}{4}J\frac{5\gamma-1}{\gamma-1}\frac{k_b}{m_i}(T_e+T_i). \tag{34}$$

On the other hand, we know that

$$a_{qu} = \frac{A_{qu}}{ln\Lambda} = \left(\frac{2m_i}{3k_b\rho}\right)(20)\left(\frac{2}{\pi}\right)^{\frac{3}{2}}\frac{4Z^2m_e^{\frac{1}{2}}k_b^{\frac{5}{2}}}{3\hbar^2 ln\Lambda} \tag{35}$$

exploiting the stages used in deriving Eq. (28) and repeating the same steps, especially with renewed Eq. (34) at this time, we can obtain a modified differential equation linking the ion temperature to the electron temperature as follows:

$$\frac{\mathrm{d}T_i}{\mathrm{d}T_e} = \frac{16}{9}\frac{A_{\mathrm{qu}}}{BJ^2}\frac{\gamma-1}{5\gamma-1}\left(\frac{m_i}{k_b}\right)^3 p_1^2\frac{T_e(T_e-T_i)}{(T_e+T_i)^2} \tag{36}$$

Multiplying this equation by, we have:

$$\frac{\mathrm{d}T_i}{\mathrm{d}x} = \frac{16}{9}\frac{A_{\mathrm{qa}}}{BJ^2}\frac{\gamma-1}{5\gamma-1}\left(\frac{m_i}{k_b}\right)^3 p_1^2\frac{T_e(T_e-T_i)}{(T_e+T_i)^2}\frac{\mathrm{d}T_e}{\mathrm{d}x}. \tag{37}$$

As it can be seen from Eq. (37), determination of improved ion temperature depends also on choosing the modified electron temperature in this equation. Consequently, we can take the gradient of electron temperature into account in Eq. (37):

$$\frac{\mathrm{d}T_e}{\mathrm{d}x} = -\frac{T_{oe}}{\left[1+\left(\frac{7}{2}\right)\frac{a_{qu}T_{oe}^{3/2}}{L_o^2}t\right]^{6/7}}\frac{x}{L_o^2}\left(1-\frac{x^2}{\frac{4L_o^2}{3}\left[1+\left(\frac{7}{2}\right)\frac{a_{qu}T_{oe}^{3/2}}{L_o^2}t\right]^{4/7}}\right)^{-1/3}, \tag{38}$$

while in [17] the ion temperature has been obtained based on the following equations:

$$\frac{\mathrm{d}T_i}{\mathrm{d}x} = \frac{16}{9}\frac{A_{cl}}{BJ^2}\frac{\gamma-1}{5\gamma-1}\left(\frac{m_i}{k_b}\right)^3 p_1^2 \frac{(T_e-T_i)}{(T_e+T_i)^2}\frac{\mathrm{d}T_e}{\mathrm{d}x} \tag{39}$$

$$T = \left(\frac{Q^2}{at}\right)^{\frac{2}{9}} f(\xi) \tag{40}$$

$$\frac{\mathrm{d}T_e}{\mathrm{d}x} = \frac{-2}{9}\left(\frac{Q^2}{a_{cl}t}\right)^{2/9}\left[\frac{5\xi_o^2}{18}\left(1-\frac{x^2}{\left(Q^{\frac{5}{2}}a_{cl}t\right)^{\frac{4}{9}}\xi_o^2}\right)\right]^{-\frac{3}{5}}\frac{1}{\left(Q^{\frac{5}{2}}a_{cl}t\right)^{\frac{2}{9}}}. \tag{41}$$

In addition, in order to make it more clear, we have reviewed the numerical results of the electron and ion temperatures determined by Eqs. (39) to (41) in Fig. 3.

Here, we have used the renewed Eqs. (17), (37), (38) modified by quantum EICF in calculating our numerical results for the ion and electron temperatures. Moreover, we have determined the temperatures at time t = 7.8, 3.3, 1.5, 0.8, 0.43, 0.23, 0.12, and 0.08 ns. The obtained curves for the electron and the ion temperatures by mentioned equations are presented in Figs. 4 and 5.

In order to have more accuracy, we have extracted the ion temperature curves plotted in Fig. 4 and put them into another figure as follows:

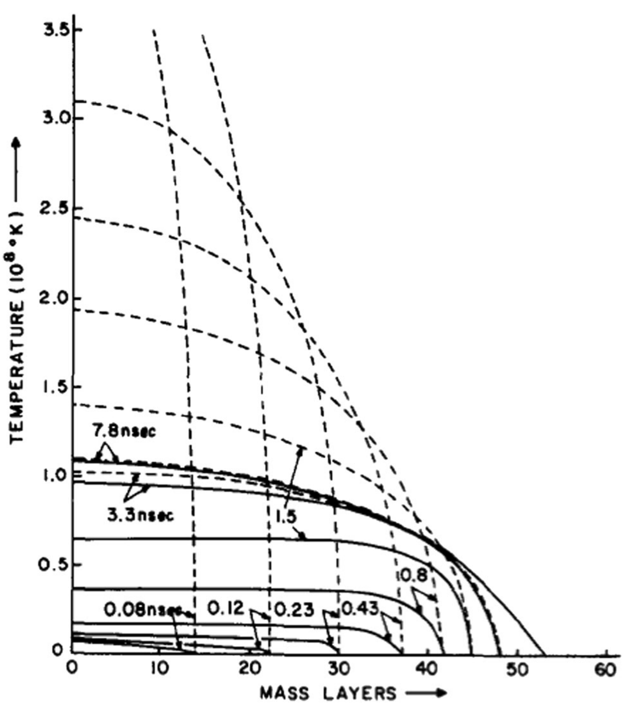

Fig. 3 The figure presented in Ref. [17] which presents the electron and ion temperature profiles at different times with classical EICF. The dotted curves refer to electron temperature; the solid curves are ion temperature. Input energy of laser beam is $E = 6.4 \times 10^{15}$ erg/cm^2. In numerical computations of Ref. [17], the medium has been divided up into mass layers. For their case, each layer $\sim$ 0.04 cm

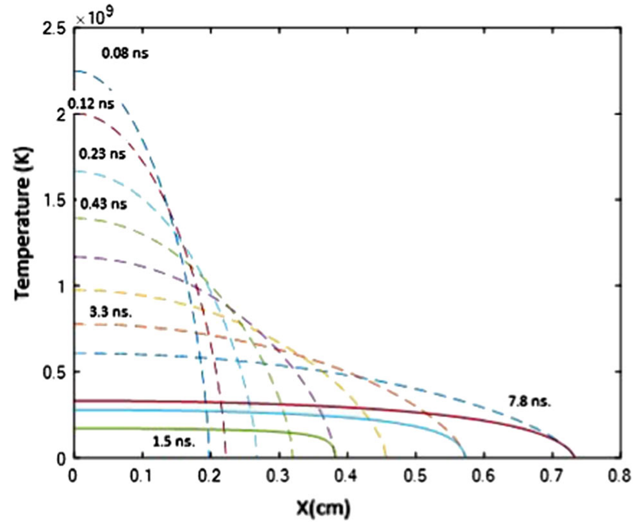

Fig. 4 Computed electron and ion temperatures profiles at different times modified by quantum EICF. The dotted curves refer to the electron temperature; the solid curves are the ion temperature. Input energy of the laser beam is $E = 6.4 \times 10^{15}$ erg/cm^2

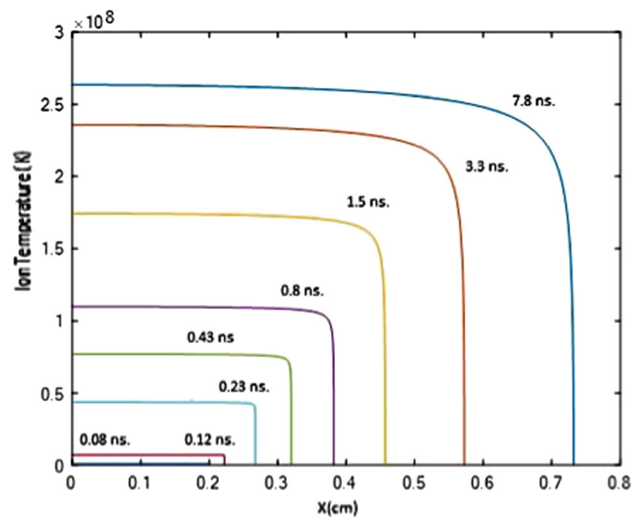

Fig. 5 Computed ion temperature profiles at different times modified by quantum EICF. Input energy of laser beam is $E = 6.4 \times 10^{15}$ erg/cm^2

Comparing Figs. 5 and 3, it was concluded that the ion temperature at t = 7.8 ns increases from (°K) to (°K). With this result, it seems that the inclusion of quantum EICF in Eq. (37) causes the ignition condition to be improved and consequently decreases the threshold energy of the ignition.

6. Conclusions

It is well known that the ion temperature profile is useful to investigate the thermonuclear reactions produced by the laser beam since the fusion burn rate depends very sensitively on the ion temperature [34]. In the case that the electron temperature is greater than the critical temperature, approximately 37.9 eV for DT plasma, it is necessary for theoretical models to use the quantum EICF. Moreover, results of Ref. [21] indicate that the quantum modified EICF affects the thermal conductivity coefficient and the thermal waves. Our theoretical analysis reveals that having a quantum insight to EICF generates a different equation for linking the ion and the electron temperatures from the one presented in [1]. As an important result, the ignition threshold equations can be renewed due to the quantum modified heat conduction coefficient. We have also found that this modification in the ignition threshold equation changes the condition of ignition and updates the numerical results investigated in the literature [17, 21]. Our numerical results for the ion temperature indicate that input energy of $E = 6.4 \times 10^{15}$ erg/cm^2 at $t = 7.8$ ns and $x = 0$ with the quantum modified EICF, discussed in Sect. 5, leads to whose value is twice greater than the value calculated in Ref. [14]. Numerical results presented in plots of the ion and electron temperature curves (Fig. 6) show that the equilibration of the ion and the electron temperature occurs at $t = 10$ ns, which is larger than the time presented in Ref. [17]. Considering the physics of energy transporting of the hot electrons, we have concluded that the quantum EICF increases the difference of the ion and electron temperatures at initial moments. Moreover, it leads to longer times for the electrons and ions to exchange energy by means of many collisions to reach the equilibrium temperature. Specifically, this claim is confirmed by Eq. (21) for since

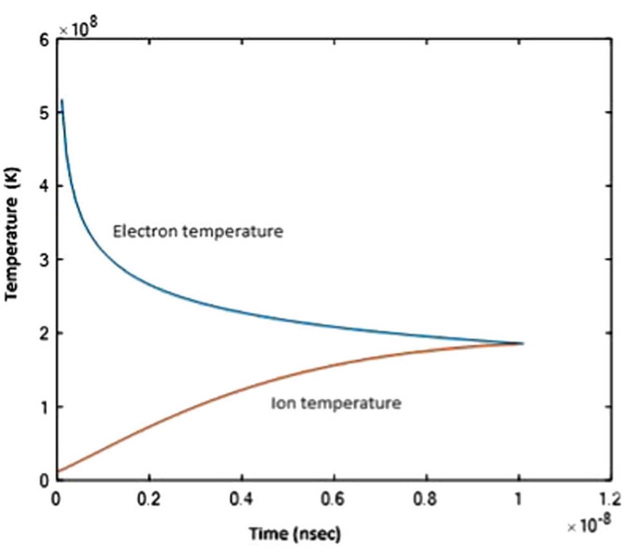

Fig. 6 Computed time history of the electron and ion temperatures in DT plasma at $x = 0$. Input energy of laser beam is $E = 6.4 \times 10^{15}$ erg/cm^2. The equilibration of ion and electron temperature occurs at $t = 10$ ns

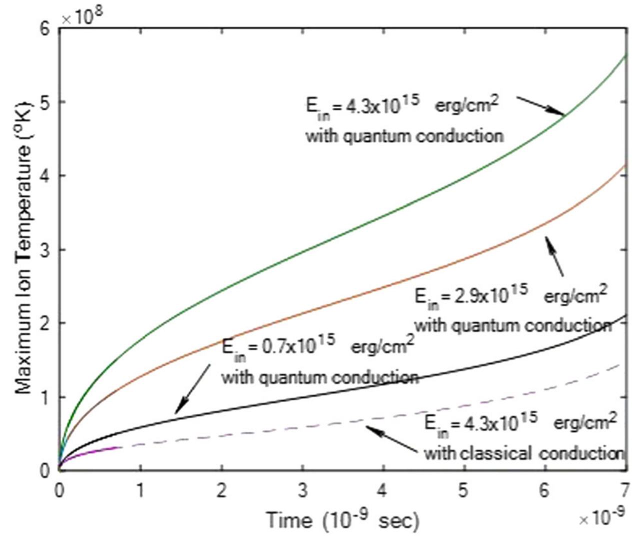

Fig. 7 Diagram of maximum ion temperature of DT plasma with respect to input energy of the laser beam. In these calculations, only one-half of particle energy is absorbed by the DT plasma. The dotted curve with a threshold energy of $E_{in} = 4.3 \times 10^{15}$ erg/cm^2 and computed with classical EICF which has been presented in Ref. [17]. The solid curve with input energy of $E_{in} = 0.7 \times 10^{15}$ erg/cm^2 gives the threshold energy of laser beam improved via quantum EICF. Other solid curves refer to ignited cases with quantum EICF

the increase in electron temperature enhances the characteristic time and this issue, in turn, increases the equilibrium time. The numerical result of the ignition threshold energy modified by quantum EICF indicates that the ignited case for DT plasma is available with the input energy as $E_{in} = 0.7 \times 10^{15}$ erg/cm^2. This research updated the conditions of Chu's side-on laser ignition of the solid DT.

Appendix: Expansion energy of plasma electrons with quantum modified EICF consideration

The expansion energy of the electrons due to ordinary Zeldovich solution for $\boldsymbol{T_e}$

The basic formula associated with the expansion energy of plasma has been obtained in [7] as follows:

$$w_{\text{expansion}} = \frac{4}{3} T \frac{\partial u}{\partial x}. \quad (42)$$

Here, T and u are the electron temperature and local velocity of fluid in plasma, respectively. The velocity u can be related to temperature T via the following formula:

$$u = C\sqrt{\frac{k_b T}{m_e}} \quad (43)$$

where C is another constant which can be calculated by comparison. However, depending on the solution of non-linear heat transfer equation, $T_e(x, t)$, we can obtain the relation of expansion energy with respect to the electron temperature. First, as a review, we have derived the expansion energy of the electrons which satisfy the non-linear heat transfer equation due to the Chu's solution.

As we see in [17], Chu's solution can be presented as follows:

$$T_e(x,t) = \left(Q^2/\mathrm{at}\right)^{\frac{2}{9}} f(\xi) = \left(Q^2/\mathrm{at}\right)^{\frac{2}{9}} \left[\frac{5}{18}\left(\xi_o^2 - \xi^2\right)\right]^{\frac{2}{5}} \quad (44)$$

where is defined as. Considering Ref. [7], the expansion energy of the hot electrons can be obtained by

$$\begin{aligned} w_{\text{expansion}} &= \frac{4}{3} T \frac{\partial u}{\partial x} = C \frac{4}{3} T \frac{\partial}{\partial x}\left(\sqrt{\frac{k_b T}{m_e}}\right) \\ &= C \frac{1}{2}\frac{4}{3} T \sqrt{\frac{k_b}{m_e T}} \frac{\partial T}{\partial x} = C \frac{2}{3} \sqrt{\frac{k_b T}{m_e}} \frac{\partial T}{\partial x}. \end{aligned} \quad (45)$$

Substituting T_e from Chu's solution, we can establish a relation as derived in Ref. [17]. After some calculations for getting the derivative of temperature T_e with respect to x, we have:

$$\begin{aligned} w_{\text{expansion}} &= \frac{4}{3} T \frac{\partial u}{\partial x} = C \frac{2}{3} \sqrt{\frac{k_b T}{m_e}} \frac{\partial T}{\partial x} = C \frac{2}{3} \sqrt{\frac{k_b T}{m_e}} \frac{\partial \xi}{\partial x} \frac{\partial T}{\partial \xi} \\ w_{\text{expansion}} &= C \frac{2}{3} \sqrt{\frac{k_b T}{m_e}} \frac{\partial \xi}{\partial x} \frac{\partial T}{\partial \xi} = C \frac{2}{3} \sqrt{\frac{k_b T}{m_e}} \left(Q^2/\mathrm{at}\right)^{\frac{2}{9}} \\ &\times \frac{x}{\left(Q^{5/2}\mathrm{at}\right)^{\frac{4}{9}}} \left(\frac{2}{5}\right)\left(-\frac{10}{18}\right)\left[\frac{5}{18}\left(\xi_o^2 - \xi^2\right)\right]^{-\frac{3}{5}} \end{aligned} . \quad (46)$$

Here, we use the chain rule as ξ is a function of x:

$$\begin{aligned} w_{\text{expansion}} &= \frac{4x}{27} C \sqrt{\frac{k_b T_e}{m_e}} \frac{\left(Q^2/\mathrm{at}\right)^{\frac{2}{9}}}{\left(Q^{5/2}\mathrm{at}\right)^{\frac{4}{9}}} \left[\left(\frac{T_e}{\left(Q^2/\mathrm{at}\right)^{\frac{2}{9}}}\right)^{\frac{5}{2}}\right]^{-\frac{3}{5}} \\ &= \frac{4}{27} \frac{1}{a} \left(\frac{x}{t}\right) C \sqrt{\frac{k_b T_e}{m_e}} T_e^{-\frac{3}{2}} = \frac{4}{27} C^2 \frac{1}{a} \frac{k_b}{m_e} T_e^{-\frac{1}{2}} \end{aligned} \quad (47)$$

$$w_{\text{expansion}} = \frac{8}{9} \frac{1}{a} \frac{k_b}{m_e} \frac{1}{T_e^{\frac{1}{2}}} = \frac{4}{27} C^2 \frac{1}{a} \frac{k_b}{m_e} T_e^{-\frac{1}{2}}. \quad (48)$$

Comparing the derived equation with the expansion energy term presented in Ref. [17] and equating it to Eq. (48), we obtain:

$$C^2 = 6 \quad (49)$$

The expansion energy of the electrons via Mayer solutions modified by quantum consideration to heat transfer equation coefficient $\boldsymbol{T_e}$

In Ref. [21], the solution of the nonlinear heat equation with quantum consideration in the ion–electron collision frequency has been derived as:

$$\begin{aligned} T_e(x,t) = &\frac{T_{\text{oe}}}{\left[1 + \left(\frac{7}{2}\right) \frac{a_{\text{qu}} T_{\text{oe}}^{3/2}}{L_o^2} t\right]^{2/7}} \\ &\times \left[1 - \frac{x^2}{\frac{4}{3} L_o^2 \left[1 + \left(\frac{7}{2}\right) \frac{a_{\text{qu}} T_{\text{oe}}^{3/2}}{L_o^2} t\right]^{4/7}}\right]^{2/3}, \end{aligned} \quad (50)$$

where T_{oe} is the initial temperature of electron and is determined by the total absorbed energy of laser at $t = 0$. Since we select neodymium glass laser as the driver laser, $L_o \approx 1.06 \times 10^{-4}$ cm. The quantity a_{qu} is defined in $\chi = a_{\text{qu}} T_e^{3/2}$, where χ is the quantum modified heat transfer coefficient. Again, by applying Eq. (45) and substituting T_e from (48), we have:

$$\begin{aligned} w_{\text{expansion}} &= C \frac{2}{3} \sqrt{\frac{k_b T_e}{m_e}} \frac{\partial T_e}{\partial x} = \\ &C \frac{2}{3} \sqrt{\frac{k_b T_e}{m_e}} \frac{\partial}{\partial x} \left[\begin{array}{c} \dfrac{T_{\text{oe}}}{\left[1 + \left(\frac{7}{2}\right) \frac{a_{\text{qu}} T_{\text{oe}}^{3/2}}{L_o^2} t\right]^{2/7}} \times \\ \left[1 - \dfrac{x^2}{\frac{4}{3} L_o^2 \left[1 + \left(\frac{7}{2}\right) \frac{a_{\text{qu}} T_{\text{oe}}^{3/2}}{L_o^2} t\right]^{4/7}}\right]^{2/3} \end{array} \right]. \end{aligned} \quad (51)$$

Taking the derivative into account with respect to x gives:

$$\begin{aligned} w_{\text{expansion}} = C \frac{2}{3} \sqrt{\frac{k_b T_e}{m_e}} &\frac{T_{\text{oe}}}{\left[1 + \left(\frac{7}{2}\right) \frac{a_{\text{qu}} T_{\text{oe}}^{3/2}}{L_o^2} t\right]^{2/7}} \\ &\times \left[- \frac{2x}{\frac{4}{3} L_o^2 \left[1 + \left(\frac{7}{2}\right) \frac{a_{\text{qu}} T_{\text{oe}}^{3/2}}{L_o^2} t\right]^{4/7}} \right] \\ &\times \left[1 - \frac{x^2}{\frac{4}{3} L_o^2 \left[1 + \left(\frac{7}{2}\right) \frac{a_{\text{qu}} T_{\text{oe}}^{3/2}}{L_o^2} t\right]^{4/7}}\right]^{-1/3} . \end{aligned} \quad (52)$$

Consequently, we have:

$$w_{\mathrm{expansion}} = C\frac{2}{3}\sqrt{\frac{k_bT_e}{m_e}}(-2x)\frac{T_{\mathrm{oe}}}{\frac{4}{3}L_o^2\left[1+\left(\frac{7}{2}\right)\frac{a_{\mathrm{qu}}T_{\mathrm{oe}}^{3/2}}{L_o^2}t\right]^{6/7}} \times \left[1-\frac{x^2}{\frac{4}{3}L_o^2\left[1+\left(\frac{7}{2}\right)\frac{a_{\mathrm{qu}}T_{\mathrm{oe}}^{3/2}}{L_o^2}t\right]^{4/7}}\right]^{-1/3}. \quad (53)$$

Considering Eq. (50), we conclude that:

$$\left[1-\frac{x^2}{\frac{4}{3}L_o^2\left[1+\left(\frac{7}{2}\right)\frac{a_{\mathrm{qu}}T_{\mathrm{oe}}^{3/2}}{L_o^2}t\right]^{4/7}}\right] = \left[\frac{T_e}{T_{\mathrm{oe}}}\left[1+\left(\frac{7}{2}\right)\frac{a_{\mathrm{qu}}T_{\mathrm{oe}}^{3/2}}{L_o^2}t\right]^{2/7}\right]. \quad (54)$$

Therefore, we have:

$$w_{\mathrm{expansion}} = C\frac{2}{3}\sqrt{\frac{k_bT_e}{m_e}}(-2x)\frac{T_{\mathrm{oe}}}{\frac{4}{3}L_o^2\left[1+\left(\frac{7}{2}\right)\frac{a_{\mathrm{qu}}T_{\mathrm{oe}}^{3/2}}{L_o^2}t\right]^{6/7}} \times \left[\frac{T_e}{T_{\mathrm{oe}}}\left[1+\left(\frac{7}{2}\right)\frac{a_{\mathrm{qu}}T_{\mathrm{oe}}^{3/2}}{L_o^2}t\right]^{2/7}\right]^{-1/2}. \quad (55)$$

As it can be seen from the numerical computations, we are able to use the following equation in Eq. (54) with sufficient accuracy:

$$\left[1+\left(\frac{7}{2}\right)\frac{a_{\mathrm{qu}}T_{\mathrm{oe}}^{3/2}}{L_o^2}t\right]^{2/7} \cong \left(\left(\frac{7}{2}\right)\frac{a_{\mathrm{qu}}T_{\mathrm{oe}}^{3/2}}{L_o^2}t\right)^{2/7}. \quad (56)$$

Finally, we have:

$$w_{\mathrm{expansion}} = C\frac{2}{3}\sqrt{\frac{k_bT_e}{m_e}}(-2x)\frac{T_{oe}}{\frac{4}{3}L_o^2\left[1+\left(\frac{7}{2}\right)\frac{a_{\mathrm{qu}}T_{\mathrm{oe}}^{3/2}}{L_o^2}t\right]^{6/7+1/7}} \times \left[\frac{T_e}{T_{\mathrm{oe}}}\right]^{-1/2} \quad (57)$$

$$w_{\mathrm{expansion}} = C\frac{2}{3}\sqrt{\frac{k_bT_e}{m_e}}(-2x)\frac{T_{oe}^{3/2}T_e^{-1/2}}{\frac{4}{3}L_o^2\left[\left(\frac{7}{2}\right)\frac{a_{qu}T_{oe}^{3/2}}{L_o^2}t\right]}. \quad (58)$$

By substituting $C^2 = 6$ in Eq. (58) and assuming, we obtain:

$$\begin{aligned} w_{\mathrm{expansion}} &= -C\frac{2}{7}\frac{1}{a_{\mathrm{qu}}}\sqrt{\frac{k_b}{m_e}}\left(\frac{x}{t}\right) \\ &= -C\frac{2}{7}\frac{1}{a_{\mathrm{qu}}}\sqrt{\frac{k_b}{m_e}}\left(C\sqrt{\frac{k_bT_e}{m_e}}\right) \\ &= -\frac{2C^2}{7}\frac{1}{a_{\mathrm{qa}}}\left(\frac{k_b}{m_e}\right)T_e^{\frac{1}{2}} \end{aligned} \quad (59)$$

$$w_{\mathrm{expansion}} = -\frac{12}{7}\frac{1}{a_{\mathrm{qa}}}\left(\frac{k_b}{m_e}\right)T_e^{\frac{1}{2}}. \quad (60)$$

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