

# Theoretical comparison analysis of long and short external cavity semiconductor laser

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**ABSTRACT**—In this paper, considering optical feedback as an optical injection, and taking in to account round-trip time role in the external cavity, a standard small signal analysis is applied on laser rate equations. By considering the relaxation oscillation ( $f_2$ ) and external cavity frequencies ( $f$ ) ratio for semiconductor laser, field amplitude response gain, optical phase and carrier number for long external cavities (LEC) and short external cavities (SEC) are obtained. Laser output intensity and resonance peak dynamics have been shown by bifurcation diagrams. Furthermore, the effects of some control parameters, such as; enhancement factor, pumping current and feedback strength, on response gain have been discussed in short and long external cavities. As a result, in optical injection, for SEC, compared to LEC, more varied dynamics are observed. Also, higher values of the response gain peak in SEC, in comparison with LEC, make SEC to be affected more by the injected beam. SEC provides greater bandwidth, and also better performance in the range of  $f \leq f_r$  compared to LEC.

**KEYWORDS:** Semiconductor lasers, External cavity, Small signal analysis, Optical injection

## I. INTRODUCTION

Semiconductor lasers (SL) are strongly sensitive against optical feedback [1-3]. Optical feedback in SL can be utilized for two purposes; first the injection signal can induce the instabilities of the laser intensity [4, 5]. Second, it can be utilized to improve the frequency stability, modifying the bandwidth and decreasing the noise and distorting characteristics [6-8]. The importance of optical feedback process in technical applications has caused the external cavity semiconductor

lasers to be a subject of wide research [9-13]. In many feasible applications, such as fiber couplers, external cavities are only a few cm. So a decrease in the lengths of external cavity ( $L_{ext}$ ) has main physical subsequences [14].  $L_{ext}$  can be specified by the chosen external cavity round-trip time ( $\tau$ ). Various theoretical and experimental studies about optical feedback effects carried out [15, 16]. Obtained results in [17], are based on comparing two basic system frequencies, relaxation oscillation and external cavity frequency ratios which presents information about feedback phase and dynamics influences on short and long time scales. Furthermore in previous studies, from dynamical systems point of view,  $\tau$  in external cavity is considered as an initial constant value. Now, by considering optical feedback as an optical injection and applying small signal analysis in laser rate equations, we can obtain comprehensive information about the capability of the maximum modulation and the modulation efficiency near relaxation oscillation frequency. In this paper by applying a standard “small-signal analysis” we obtained a response gain of amplitude, optical phase, and carrier numbers for long external cavity (LEC) and short external cavity (SEC) using laser rate equations [17, 18]. SL dynamical behavior has been analyzed through bifurcation diagrams versus frequency as control parameter for LEC and SEC regimes. Also, in a given frequency domain, we have reported response gain maximum values (resonance peak ( $f=f_r$ )) for specific amounts of enhancement factor  $\alpha$ , pumping current  $P$  and feedback strength  $k$ . When the relaxation oscillation frequency becomes close to the

external cavity frequency an intensive mode competition between the two frequencies occurs, and an external cavity mode suppresses the excitation of the relaxation oscillation [20]. In fact, if one just depicts bifurcation and time series diagrams which have been mentioned in prior works [14-16], lots of important details will be missed. The obtained results in [14, 16], are base on system internal frequencies, number of modes, influence of feedback phase and dynamics on time scales for short round-trip times ( $\tau$ ). So, our principle goal is studying SL behavior, by driven response variations in field amplitude, optical phase and carrier numbers regarding external cavity frequency and  $x(x = \alpha, P, k)$  simultaneously, as a control parameter for LEC and SEC. This allows us to present a consistent overall picture of dynamics while we are considering external cavity type. It should be noted that resonance peak frequency specifies the susceptibility of the maximum modulation in semiconductor laser and the larger values for the response gain peak causes laser have better performance in the optical injection process.

## II. MODEL

Lang and Kobayashi (LK) equations describe the models are using extensively to explain a semiconductor laser subjected to feedback from an external cavity [18]. The LK equations can be written in the compact dimensionless form as [15]:

$$\frac{dE}{dt} = (1 + i\alpha)NE + kE(t - \tau)e^{-iC_p} \quad (1)$$

$$T \frac{dN}{dt} = P - N - (1 + 2N)|E|^2 \quad (2)$$

For complex electric field  $E$  and inversion parameter  $N$ ,  $\alpha$  is line width enhancement factor,  $C_p$   $2\pi$ - periodic feedback phase,  $P$  pump current and  $K$  is the feedback strength. In these equations, time is normalized against cavity photon lifetime (1ps) and  $T$  is carrier lifetime (1ns) to the photon lifetime ratio [14]. External round trip time  $\tau$  is also normalized

against photon lifetime. We performed our studies on a short external cavity and a long external cavity. In our numerical investigation values for external short and long cavities are considered  $\tau=70$  and  $\tau=700$  respectively. For short external cavity different parameters are held fixed at  $T=1710$ ,  $P=0.8$  and  $\alpha=5.0$  [16] and for long one these values are  $T=300$ ,  $P=0.001$  and  $\alpha=6$  [19]. We discussed retard differential equations derived by Lang and Kobayashi which describes model for external cavity semiconductor lasers. These equations explain the evolution of complex electric field amplitude  $E(t) = A(t)\exp(i\phi(t))$  and the carrier  $N(t)$ . The LK equations can be rewritten as follows [18]:

$$\frac{dA(t)}{dt} = N(t)A(t) + kA(t - \tau)\cos(\phi(t) - \phi(t - \tau) + C_p) \quad (3)$$

$$\frac{d\phi(t)}{dt} = \alpha N(t) - k \frac{A(t - \tau)}{A(t)} \sin(\phi(t) - \phi(t - \tau) + C_p) \quad (4)$$

where  $A$  is the slow varying field amplitude,  $\phi$  the optical phase and  $N$  is the carrier number.  $\phi(t) - \phi(t - \tau)$  is phase retardation during external cavity round-trip time  $\tau$  which  $\tau = 2L_{ext}/c$  represents round-trip retard time within external cavity, where  $c$  is the velocity of light in free space. In most studies, the length of long external cavities is from 10 cm to several meters [16] and the short external cavity length is something between 1 and 7.5 cm [14].

## III. ANALYSIS OF LK EQUATIONS AND RESPONSE GAIN DERIVATION

General lasers can be considered as damped nonlinear oscillators which are exhibiting damping oscillation [18]. The relaxation oscillation and external cavity frequencies play important role in the bifurcation diagrams. External cavity frequency is the frequency which is determined by the round-trip time in the external cavity [16]. When the relaxation oscillation frequency gets close to the external cavity frequency an intensive competition mode happen between these two frequencies

and an external cavity mode suppresses the relaxation oscillation excitation (resonance) [20]. Also modulation performance is increased near the relaxation oscillation frequency. In this section, linear stability [17] of the retarded rate equations are analyzed considering the external cavity round trip time  $\tau$  role in the LK equations (the external cavity frequency changes with  $L_{ext}$  variations). Presented analyze is a useful method to attain physical insight toward dynamical behavior of a semiconductor laser with optical feedback. The stationary solutions of Eqs. (2) to (4), is as follow:

$$A_{st}^2 = \frac{P - N_{st}}{1 + 2N_{st}} \quad (5)$$

$$\Delta N_{st} = N_{st} - N_{th} = -k \cos(\varphi(t) - \varphi(t - \tau) + C_p) \quad (6)$$

$$w_{st} = -k(\sin(w_{st}\tau) + \alpha \cos(w_{st}\tau)) \quad (7)$$

The laser response can be considered as  $x_s(t) = x_{st} + \delta x(t)$ , where  $x_{st}$  is the steady-state value and  $\delta x(t)$  is a small perturbation, is applied to injected field steady state value ( $x=A$ ,  $\varphi$ , and  $N$ ). Here the deviation is defined as an exponential perturbation in the form of  $\delta x(t) = \delta x_0 \exp(\lambda t)$ , where  $\lambda$  represent a perturbation parameter which is a complex number [17, 18]. By substituting  $x_s(t) = x_{st} + \delta x(t)$ , in to the Eqs. (2) to (4) leads to the set of coupled equations:

$$\lambda \delta A = N_{st} \delta A + A_{st} \delta N + k \cos \Delta \varphi_{st} \delta a_{inj} - k A_{st} \sin \Delta \varphi_{st} (\delta \varphi - \delta \varphi_{inj}) \quad (8)$$

$$\lambda \delta \varphi = \alpha \delta N + \frac{k \sin \Delta \varphi_{st}}{A_{st}} \delta A - k \cos \Delta \varphi_{st} (\delta \varphi - \delta \varphi_{inj}) - k \frac{\sin \Delta \varphi_{st}}{A_{st}} \delta a_{inj} \quad (9)$$

$$T \lambda \delta N = -\delta N - 2A_{st}^2 \delta N - 2A_{st} (1 + 2N_{st}) \delta A \quad (10)$$

Using Laplace transform for the perturbation, result leads to the below matrix form:

$$\begin{bmatrix} \lambda + k \cos(w_{st}\tau) & k A_{st} \sin(w_{st}\tau) & -A_{st} \\ -k \sin(w_{st}\tau)/A_{st} & \lambda + k \cos(w_{st}\tau) & -\alpha \\ 2T^{-1}(1+2N_{st})A_{st} & 0 & \lambda + T^{-1}(1+2A_{st}^2) \end{bmatrix} \begin{bmatrix} \delta A \\ \delta \varphi \\ \delta N \end{bmatrix} = \begin{bmatrix} k A_{st} \sin(w_{st}\tau) \delta \varphi_{inj} + k \cos(w_{st}\tau) \delta a_{inj} \\ -k \sin(w_{st}\tau)/A_{st} \delta a_{inj} + k \cos(w_{st}\tau) \delta \varphi_{inj} \\ 0 \end{bmatrix} \quad (11)$$

Now driven response in the amplitude, optical phase and carrier number can be obtained:

$$\frac{\delta A}{\delta a_{inj}} = \frac{(ak^2 - k \cos(\Delta \varphi_{st})w^2) + iw(k^2 + ka \cos(\Delta \varphi_{st}))}{D(iw)} \quad (12)$$

$$\frac{\delta \varphi}{\delta \varphi_{inj}} = \frac{(ak^2 + kb^2(\cos(\Delta \varphi_{st}) - \alpha \sin(\Delta \varphi_{st})))}{D(iw)} - \frac{kw^2 \cos(\Delta \varphi_{st}) + iw(k^2 + ka \cos(\Delta \varphi_{st}))}{D(iw)} \quad (13)$$

$$\frac{\delta N}{\delta a_{inj}} = \frac{-(bk^2 + iwkb \cos(\Delta \varphi_{st}))}{A_{st}} \quad (14)$$

$$a = \frac{1}{T}(1 + 2A_{st}^2), \quad b = \frac{2}{T}(1 + 2N_{st})A_{st}^2 \quad (15)$$

where  $D$  is the determinant of the coefficients matrix which is given by:

$$D(\lambda) = \lambda^3 + (a + 2k \cos(w_{st}\tau))\lambda^2 + (b^2 + k^2 + 2ka \cos(w_{st}\tau))\lambda + ak^2 + kb^2(\cos(w_{st}\tau) - \alpha \sin(w_{st}\tau)) \quad (16)$$

Note that equations (12) to (15) are complex, and can be divided into amplitude and phase in form of  $G_x \exp(i\Theta_x)$ , ( $x = A, N, \varphi$ ) where  $G_x$  and  $\Theta_x$  represent the response gain and the phase shift [17].

#### IV. EFFECT OF THE ROUND-TRIP TIME ON THE RESPONSE GAIN

In previous section, as calculated in Eqs. 12, 13, 14 and 15, laser response in set of amplitudes, optical phase and carrier numbers can be derived simultaneously. To investigate role of  $\tau$  on lasers dynamical behavior and

response gain variations [17], we have considered two types of laser LEC and SEC cavities for semiconductor lasers.

### Long External Cavity

Figure 1.a shows response gain plots as a function of the round trip frequency of the light in the external cavity,  $\omega/2\pi$ , for LECs. In this figure, thick, thin and dotted lines correspond to the response in field amplitude ( $G_A$ ), optical phase ( $G_\phi$ ) and carrier number ( $G_N$ ), respectively.

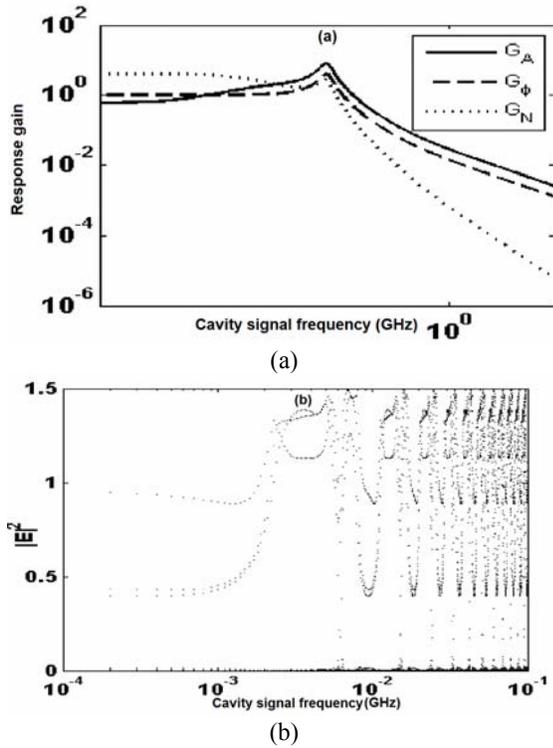


Fig. 1. a) Response gain of LEC, (Thick, thin, and dotted lines are correspond with responses in field amplitude, optical phase and carrier number, respectively), and b) Bifurcation diagram of the intensity,  $|E|^2$  versus signal frequency output,  $\omega/2\pi$  (GHz), for LEC with  $\tau=700$  and  $T=300$ .

In Fig. 1.a, all plots exhibiting a resonance with the maximum which is appearing around  $\omega/2\pi=0.07$  GHz.  $G_A$  is so low in the lower frequency range, i.e., the output signal amplitude is reduced. In long external cavity, until external cavity frequency is lower than damping frequency, laser dynamics is stable and periodic. When external cavity frequency

increased, laser settle in resonant condition, and maximum value achieve for  $G_A$ ,  $G_\phi$  and  $G_N$ . By increasing external cavity frequency, getting away from resonant frequency, output intensity dynamics transform in to chaotic dynamics.

Fig. 1.b depicts dynamics of the  $|E|^2$  as a function of frequency. As it can be seen in this bifurcation diagram, the Quasi Periodic (QP) at  $0.001 < \omega/2\pi < 0.01$  and Chaotic (CH) at  $0.01 < \omega/2\pi < 0.2$  dynamics are appeared as frequency increases, and the resonant peak dynamics is chaotic.

### Short External Cavity

The gain frequency response of the field amplitude, optical phase and carrier number in the SEC case is similar to the LEC case, as shown in Fig. 2.a. However, around resonant frequency it has been seen that response gain for short cavity laser is less than the response gain for long cavity. The response gain for amplitude and carrier number in lower frequencies, unlike LEC, is lower than its response gain in resonant frequency. In high frequency, similar frequency response for both of the external cavities is observed. SEC output intensity dynamics is shown during frequency increase as a control parameter, in Fig. 2b. The laser output bifurcation curve in terms of control parameter and frequency contains:

Period1(P1)( $0.016 < \omega/2\pi < 0.026$ ), CH( $0.026 < \omega/2\pi < 0.047$ ), QP( $0.047 < \omega/2\pi < 0.054$ ), Period7(P7)( $0.054 < \omega/2\pi < 0.056$ ), QP( $0.056 < \omega/2\pi < 0.062$ ), P1( $0.062 < \omega/2\pi < 0.11$ ), CH( $0.11 < \omega/2\pi < 0.13$ ) and QP( $0.13 < \omega/2\pi < 0.2$ ) as shown in Fig. 2c. The resonant peak in short external cavity is also QP. As it can be seen from Fig. 2, for frequencies lowers than damping frequency, laser dynamics is stable and periodic. For frequencies higher than resonant frequency, laser dynamics transform in to unstable dynamics. Generally, for both kind of cavities, when laser is settled in stable condition, (external cavity frequency  $<$  resonant frequency)  $G_N$ ,  $G_\phi$  and  $G_A$  values increase, and their unstable dynamics decreases. But in SEC, for low external cavity

frequencies,  $G_A$  and  $G_\phi$  variations, oppose LEC behavior. For conclusion, in Figs. 1.a and 2.a, when the injected signal frequency is higher than relaxation oscillation frequency, the carrier cannot pursue the modulation rate, and the modulation intensity quickly reduces with the increase of the external cavity signal frequency.

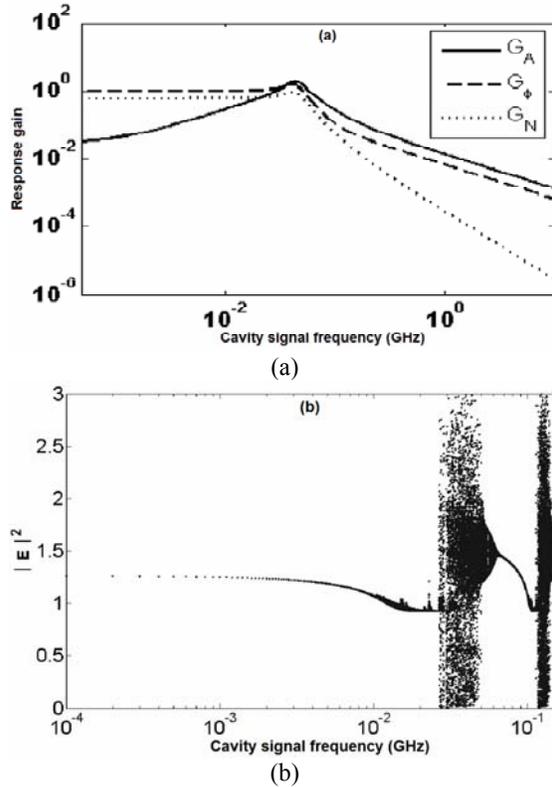


Fig. 2. a) Response gain of SEC, (Thick, thin, and dotted lines are corresponding with response in field amplitude, optical phase and carrier number, respectively), and b) Bifurcation diagram of the intensity,  $|E|^2$  versus the output signal frequency,  $\omega/2\pi$  (GHz), for SEC with  $\tau = 70$ ,  $T = 1710$ .

In order to understand  $\tau$  role we have studied influence of the chosen  $\tau$  as a fixed control parameter on the dynamics of the laser output versus  $\omega/2\pi$ . Results are shown in Fig. 3. This figure is plotted for  $T=550$  and  $\tau = 121$  [16]. As shown in Fig. 3 as frequency increases, laser output shows dynamics of: CH( $0.001 < \omega/2\pi < 0.01$ ), QP ( $0.01 < \omega/2\pi < 0.02$ ), P7( $0.02 < \omega/2\pi < 0.03$ ), P1( $0.03 < \omega/2\pi < 0.045$ ), CH( $0.045 < \omega/2\pi < 0.067$ ), QP( $0.067 < \omega/2\pi < 0.072$ ),

and P7( $0.072 < \omega/2\pi < 0.095$ ), respectively. Comparing Fig. 3, Figs. 1b and 2b, shows that by reducing stable dynamics range, unstable dynamics range increases, but, response gain and phase shift values experiences a main variations, as the  $\tau$  value increases.

In conclusion, in Figs. 1.a and 2.a, in the external cavity frequency  $<$  resonant frequency range, the response has a high gain comparable to the resonant-peak gain, that an energy transfer for the driving-signal oscillation occurs between the field amplitude and the carriers. In the external cavity frequency  $<$  resonant frequency range larger oscillation energy of the driving signal is transferred to drive the carrier oscillation, leading to the amplitude signal attenuation with smaller supply of the energy [17].

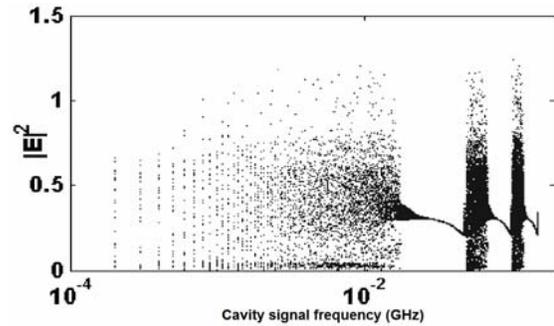


Fig. 3. Bifurcation diagram of the intensity,  $|E|^2$ , Vs. output signal frequency,  $\omega/2\pi$  (GHz), for LEC with  $\tau = 121$  and  $T = 550$ .

Something that is needed to be considered is that the field carrier coupling is intensively dependent to generation of the relaxation oscillations. Hence, the most influential coupling between them is observed at the relaxation oscillation frequency. In the external cavity frequency  $<$  resonant frequency range, the carrier coupling is forceful with its larger oscillation, whereas the coupling becomes weaker for external cavity frequency  $>$  resonant frequency. The strength of the coupling can describe the oscillation energy exchange between the amplitude and carrier [17]. Therefore, in the external cavity frequency  $>$  resonant frequency range the most

of the driving oscillation energy is shifted to the field amplitude oscillation.

## V. EFFECT OF THE $\alpha$ , $P$ , AND $k$ VARIATIONS ON RESPONSE GAIN FOR SECS AND LECs REGIMES

In previous section,  $\alpha$ ,  $P$ , and  $k$  are considered as constant. However, these quantities variations may led to changes in field amplitude gain, optical phase and carrier number. In this section, considering particular values for  $\alpha$ ,  $P$ , and  $k$ , we have discussed their effects on amplitude field response gain, in long and short external cavities regime. The plots of the response gain as a function of  $\alpha$  and  $\omega/2\pi$  for LEC and SEC lasers are shown in Fig 4.a and 4.b, respectively.

As it can be seen in Fig. 4.a, for a given frequency, the  $G_A$  increases as  $\alpha$  increases, and it reaches to a maximum value at  $\alpha=6$ . These changes are gradual for  $0<\alpha<1.5$ , and intensive for  $1.5<\alpha<6$ . On the other hand, for a given  $\alpha$ ,  $G_A$  highest variation for LEC, occurs in  $1.5<\alpha<6$  and  $0.02<\omega/2\pi<0.1$  and it has a resonant peak in  $g(\alpha, \omega/2\pi)=g(6, 0.07)=8$ . In Fig. 4.b, for a given frequency, the amplitude gain increases as  $\alpha$  increases, and it reaches to a maximum value at  $\alpha=2.35$ , afterwards as the  $\alpha$  increases  $G_A$  declines. Amplitude gain most variation, for SECS, occurs in  $1.5<\alpha<6$  and  $0.02<\omega/2\pi<0.06$ , and the resonant peak is in  $g(\alpha, \omega/2\pi)=g(2.35, 0.04)=29$ . In summary, for response gain the LECs frequency domain is greater than SECS frequency domain, but the amplitude gain in SECS at the resonant peak is higher than LECs. In dynamical point of view, in the resonant peak, the dynamics of SECS and LECs are QP and CH, respectively. According to Eqs. 12, 13, and 14, it was found that, the variation of optical phase response gain versus  $\alpha$  and  $\omega/2\pi$  are similar to variation of  $G_A$  for SEC (Fig 4.a) and for LEC (Fig. 4.b). In addition, as shown in Figs. 5.a and 5.b the variation of  $G_N$  is different from variation of amplitude response gain in the LEC and SEC. As it can be seen in Fig 5.a, in low frequencies, for  $0.001<\omega/2\pi<0.04$ , the

response gain decreases as  $\alpha$  increases. In addition, for a given  $\alpha$ , the  $G_N$  decreases as the frequency increases and at the  $\omega/2\pi=0.04$  reaches to a minimum value. In high frequencies ( $\omega/2\pi>0.04$ ), the observed variation for the  $G_N$  is similar to  $G_A$  behaviors. But, in low frequencies,  $G_N$  is higher than the  $G_A$  in the resonant frequency. Finally, in low frequencies,  $0.002<\omega/2\pi<0.01$ ,  $G_N$  of the SEC, is constant, and for  $\omega/2\pi>0.01$  and  $0<\alpha<1$  domain the response gain decreases as frequency increases. Ultimately in  $1<\alpha<6$  domain,  $G_N$  increases as the frequency increases and reaches to a maximum value ( $G_N = 17$ ).

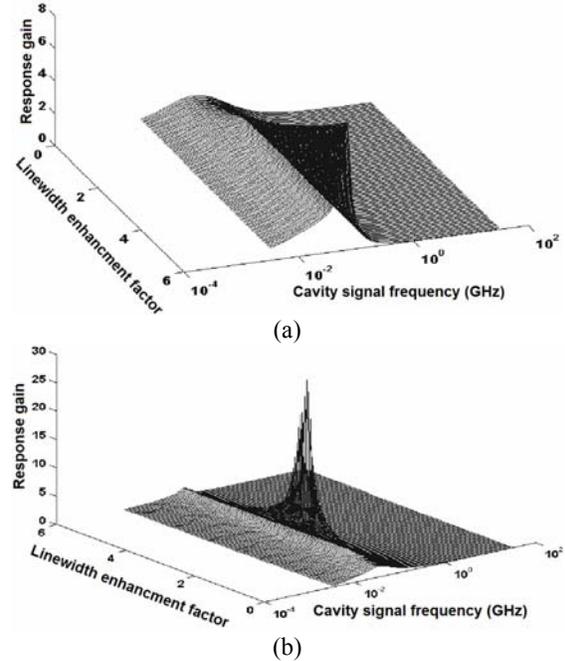


Fig. 4. Amplitude response gain against  $\alpha$  and  $\omega/2\pi$  (GHz) for a) LEC and b) SEC.

By comparing Figs. 4 and 5, it can be seen that for short external cavity, variations of  $G_N$  and  $G_A$  are the same, but for long external cavity, variations of  $G_N$  and  $G_A$  are quite different. Figures 6.a and 6.b show the  $G_A$  curve as a function of  $k$  and  $\omega/2\pi$  for LEC and SECS, respectively. As it can be seen from figure 6.a, most of the  $G_A$  variations have occurred in the  $0.01<k<0.044$  and  $0.01<\omega/2\pi<0.04$  domains. The resonant peak is located at

$g(k, \omega/2\pi) = g(0.021, 0.022) = 87$ . In Fig. 6.b, we observe that for a given frequency, the amplitude gain increases as  $k$  increases, and reaches to a maximum value at  $k = 0.07$ . Meanwhile, these changes are gradual for  $0 < k < 0.03$ , and intensive for  $0.03 < k < 0.07$ .

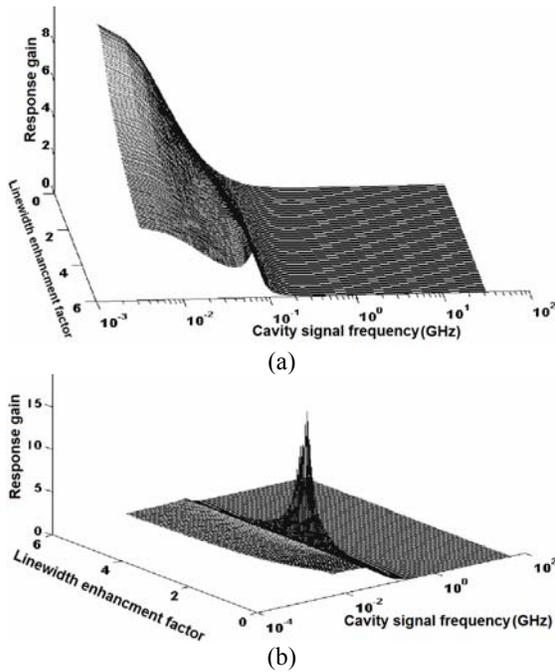


Fig. 5. Response gain of carrier number versus  $\alpha$  and  $\omega/2\pi$  (GHz) for a) LEC and b) SEC.

On the other hand, for a given  $k$ , the amplitude gain increases as the amplitude increases, and it reaches to a maximum value at  $\omega/2\pi = 0.055$ . It can be concluded That most of the variations for SEC amplitude gain occurs at  $0.03 < k < 0.07$  and  $0.01 < \omega/2\pi < 0.1$ , and the resonant peak is in  $g(k, \omega/2\pi) = g(0.07, 0.055) = 5.2$ . Comparing Figs. 6.a and 6.b, shows that SECs frequency bandwidth is greater than LECs one; however, the amplitude gain at resonant peak of the LEC is greater than of the SEC. In dynamical point of view, the observed dynamics for the resonant peaks of SEC and LEC are P1 and QP, respectively. Figures 7.a and 7.b show the plots of the response gain as a function of  $P$  and  $\omega/2\pi$  for LECs and SECs, respectively.

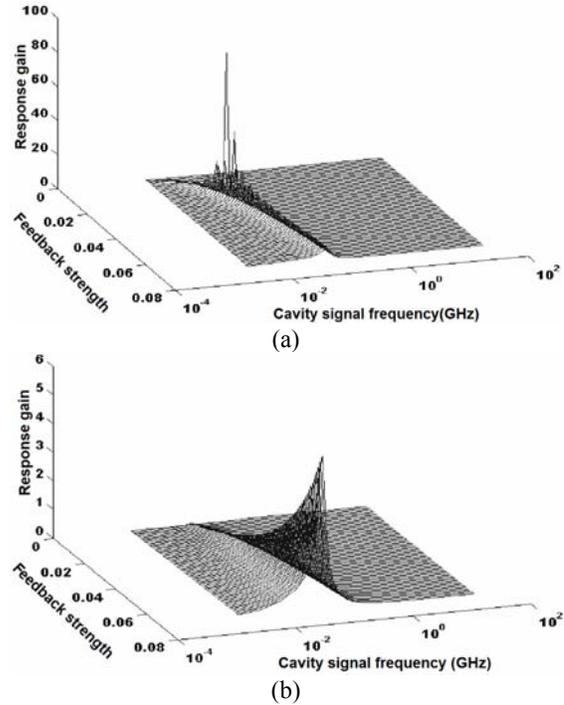


Fig. 6. Response gain of amplitude versus  $k$  and  $\omega/2\pi$  (GHz) for (a) LEC and (b) SEC.

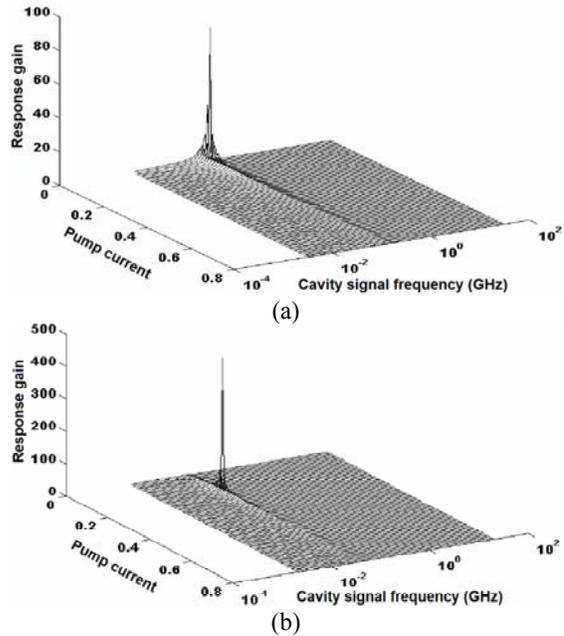


Fig. 7. Response gain of amplitude versus  $P$  and  $\omega/2\pi$  (GHz) for (a) LEC and (b) SEC.

The response gain amplitude of LEC has a resonant peak at  $g(P, \omega/2\pi) = g(0.04, 0.073) = 100$ , (Fig. 7.a). While the amplitude SEC response gain has a resonant peak at  $g(P, \omega/2\pi) = g(0.2, 0.03) = 417$ , (Fig. 7.b). At the mean time, the behavior of optical phase and

carrier number gains, for the LEC, SEC, is similar to Figs 7.a and 7.b, respectively. By considering that the larger values for the response gain peak causes laser have a better performance in the optical injection process, the obtained results in this section show that the LEC has a better performance in large values of  $\alpha$  (Figs. 4 and 5),  $k$  (Fig. 6) and small values of  $P$  (Fig 7). But in comparison with LEC SEC has a better performance in small values of  $\alpha$  and large values of  $P, k$ .

Meanwhile, the SEC has a better performance in strong injection regimes (high values of  $k$ ). In other words, in optical injection process (from external mirror), for SEC, compared to the LEC, the greater value can be obtained for response gain peaks. Thus, short cavities can be affected more by the injected beam (see Fig. 6).

## VI. CONCLUSION

In this paper, results show that by increasing  $\tau$  values, the unstable dynamics range is increased and response gain values experience considerable variations. It was shown that as  $\tau$  values increases, the unstable dynamics range increases and response gain values experience high variations.

Also, by comparing the bifurcation results obtained from output intensity analysis against frequency and diagrams of the response gain versus frequency, it can be seen that the dynamics of the resonant peak for SEC is quasi periodic and for LEC is chaotic. As a result in coupled systems based on optically injected semiconductor lasers, an increase in  $\tau$  value leads to the stability reduction in synchronization process. This can be seen, from the response gain figures and types of the resonance peaks dynamics, that the stable dynamics frequency domain in the SEC is greater than one LECs one. Furthermore, we have investigated effects of  $\alpha, P$ , and  $k$  values variation on response gain of the amplitude, optical phase and carrier numbers for LEC and SEC. based of this analysis, following results can be concluded:

In short external cavity, by increasing the  $\alpha$  value, the variations of  $G_N$ , are in the same range of  $G_A$  variations. Only for a particular  $\alpha$ , highest value of response gain obtained for resonant frequency. Variations of  $G_N$  and  $G_A$  versus  $\alpha$  and  $\omega/2\pi$  are in similar range. For low external cavity frequencies  $G_A$  and  $G_\phi$  variations are not equivalent. SEC has better performance in small amounts of  $\alpha$  and large amounts of  $P, k$ . The greater values of the response gain peaks in SEC, are causing SEC to be affected more by the injected beam. The highest amount of the response gain can be obtained for SEC in  $(\alpha, k, P) = (2.35, 0.07, 0.2)$ .

In long external cavity, by increasing the  $\alpha$  value, response gain in resonant frequency also increases. Variations of  $G_N$  and  $G_A$  are not in same range. For low external cavity frequencies,  $G_A$  and  $G_\phi$  variations, are equivalent. LEC has a better performance in high values of  $\alpha, k$  and low values of  $P$ . Highest amount of the response gain can be obtained for LEC in  $(\alpha, k, P) = (6, 0.021, 0.04)$ .

Therefore, it can be stated that the SEC semiconductor lasers provide greater bandwidth respect to LEC semiconductor lasers in the transmitter and receiver systems, and also they have better performance in the range of external cavity frequency  $\leq$  relaxation oscillation frequency.

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