

Band structure of one-dimensional doped photonic crystal with three level atoms using the Fresnel coefficients method

A. Jafari*, A. Rahmat[†] and S. Bakkeshizadeh[‡]

*Department of Physics, Faculty of Science,
Urmia University, Urmia, P. O. Box 165, Iran*

**a.jafari@urmia.ac.ir*

†a.rahmatnezamabad@urmia.ac.ir

‡s.bakkeshizadeh@urmia.ac.ir

Received 4 February 2017

Revised 19 June 2017

Accepted 5 July 2017

Published 8 August 2017

We consider a one-dimensional photonic crystal (1DPC) composed of double-layered dielectrics. Electric permittivity and magnetic permeability of this crystal depends on the incident electromagnetic wave frequency. We suppose that three level atoms have been added to the second layer of each dielectric and this photonic crystal (PC) has been doped. These atoms can be added to the layer with different rates. In this paper, we have calculated and compared the band structure of the mentioned PC considering the effect of added atoms to the second layer with different rates through the Fresnel coefficients method. We find out that according to the effective medium theory, the electric permittivity of the second layer changes. Also the band structure of PC for both TE and TM polarizations changes, too. The width of bandgaps related to “zero averaged refractive index” and “Bragg” increases. Moreover, new gap branches appear in new frequencies at both TE and TM polarizations. In specific state, two branches of “zero permittivity” gap appear in the PC band structure related to TM polarization. With increasing the amount of the filling rate of total volume with three level atoms, we observe a lot of changes in the PC band structure.

Keywords: Photonic crystals; band structure; Fresnel coefficients method; effective medium theory.

PACS numbers: 42.70.Qs, 78.20.Ci, 78.67.Pt

1. Introduction

Photonic crystals (PCs) are composed of dielectric layers with periodical spatial modulation of refractive indexes. PCs are classified into three main types:

*Corresponding author.

one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D). One-dimensional photonic crystal (1DPC) is one of the simplest types of PCs in which refractive index changes periodically in one direction. In 2D and 3D PCs, we have a periodicity of refractive index in two and three directions, respectively. 1DPCs can be made of accumulated layers. 2D PCs can be made by photolithography or by drilling holes in an appropriate substrate. The fabricated methods for 3D PCs are drilling at different angles and stacking multiple 2D layers on top of one another.¹⁻⁵ The interest in PCs bandgaps comes from their application in research and technology such as filters, waveguides, optical switches and cavities.⁶⁻¹¹ For the reason of periodicity of refractive index in PCs, the specific areas of electromagnetic field wavelength do not propagate inside the crystal and these areas are called PCs bandgaps. The PC concept was initially introduced by Yablonoitch and John in 1987.^{12,13} Metamaterials (MMs) are the new class of materials with a negative refractive index that are used in PC structures. The bandgap related to “zero averaged refractive index” appears in multilayers composed of ordinary materials with positive refractive index and MMs with negative refractive index. In most of the previous papers the band structure of different PCs has been calculated by transfer matrix method.¹⁴⁻¹⁹ In our previous paper, we calculated the band structure of an instance of 1DPCs composed of double-layered dielectrics by Fresnel coefficients method.²⁰ In this paper, we extended our calculations on a band structure at the presence of three level atoms which are added to the dielectrics second layer. The purpose of this paper is to evaluate the effect of added atoms on the PC band structure. We know that, according to the effective medium theory, the presence of atoms causes change in electric permittivity.²¹ We find out that the changes in effective electric permittivity will affect the frequency wideness of some of the gap branches in both TE and TM polarizations and creation of new gap branches in new frequencies.

This paper consists of four sections. In Sec. 2, we mention the basic considerations of Fresnel coefficients method for calculating the PCs band structure. In Sec. 3, we calculate and compare the band structure of a PC composed of double-layered dielectrics considering the effect of added atoms in the second layer of dielectrics. The paper concludes in Sec. 4.

2. Basic Consideration

For calculating the 1DPCs band structure, the transfer matrix method is often used. However, we used the Fresnel coefficients method in our paper.¹⁴⁻³² The advantage of this method is unlike the transfer matrix method which is based on the Bloch theorem, we do not need any hypothesis about the periodicity of the system. In Fresnel coefficients method, by using the refractive index of each layer and initial radiation angle on the crystal surface, the transmission and reflection coefficients of the crystal are determined. Using Eqs. (1) and (2), the necessary and sufficient conditions for entire reflection from the surface of 1DPC composed

of double-layered dielectrics can be acquired.³⁵ Therefore, we can calculate the PC band structure.³²⁻³⁵

$$\frac{(\rho_1 - \rho_2)^2}{(1 - \rho_1^2)(1 - \rho_2^2)} \sin \Delta_1 \sin \Delta_2 > \cos^2 \left(\frac{\Delta_1 + \Delta_2}{2} \right), \quad (1)$$

$$\frac{(\rho_1 - \rho_2)^2}{(1 - \rho_1^2)(1 - \rho_2^2)} \sin \Delta_1 \sin \Delta_2 < -\sin^2 \left(\frac{\Delta_1 + \Delta_2}{2} \right), \quad (2)$$

where ρ is the Fresnel reflection coefficient from the dielectric upper layer. Δ is the single-path phase shift and is defined as

$$\Delta = 2\pi \left(\frac{d}{\lambda_0} \right) \sqrt{n^2 - \sin^2 \theta}, \quad (3)$$

in which, n and d are the refractive index and thickness of each dielectric layer, respectively and θ is the initial incident angle on the crystal surface. Substituting Fresnel reflection coefficients into Eqs. (1) and (2) yields

$$G_{s,p}(n_1, n_2, \theta) \sin \Delta_1 \sin \Delta_2 > \cos^2 \left(\frac{\Delta_1 + \Delta_2}{2} \right), \quad (4)$$

$$G_{s,p}(n_1, n_2, \theta) \sin \Delta_1 \sin \Delta_2 < -\sin^2 \left(\frac{\Delta_1 + \Delta_2}{2} \right), \quad (5)$$

where

$$G_s(n_1, n_2, \theta) = \frac{1}{4} \sqrt{\frac{n_1^2 - \sin^2 \theta}{n_2^2 - \sin^2 \theta}} + \frac{1}{4} \sqrt{\frac{n_2^2 - \sin^2 \theta}{n_1^2 - \sin^2 \theta}} - \frac{1}{2}, \quad (6)$$

$$G_p(n_1, n_2, \theta) = \frac{1}{4} \left(\frac{n_2}{n_1} \right)^2 \sqrt{\frac{n_1^2 - \sin^2 \theta}{n_2^2 - \sin^2 \theta}} + \frac{1}{4} \left(\frac{n_1}{n_2} \right)^2 \sqrt{\frac{n_2^2 - \sin^2 \theta}{n_1^2 - \sin^2 \theta}} - \frac{1}{2}. \quad (7)$$

Labels s and p are related to the TE and TM polarization of incident light. Equations (4)–(7) are the fundamental relationships for calculating of the band structure of 1DPCs by Fresnel coefficients method.

3. Results and Discussions

By adding three level atoms over the dielectrics second layer, the band structure of the aforementioned PC is calculated.

Optical characteristics of this PC are as following:

$$\left. \begin{aligned} \varepsilon_0 = \mu_0 = 1, \quad \varepsilon_2(f) = 1 + \frac{5^2}{0.9^2 - f^2} + \frac{10^2}{11.5^2 - f^2}, \quad \varepsilon_1 = \mu_1 = 1 \\ \mu_2(f) = 1 + \frac{3^2}{0.902^2 - f^2}, \quad d_1 = 12 \text{ mm}, \quad d_2 = 6 \text{ mm} \end{aligned} \right\}, \quad (8)$$

where f is the incident light frequency in GHz. d_1 and d_2 are the thickness of the first and second layers of dielectrics, respectively. We have considered the optical

characteristics of the PC as Eq. (8), because at some frequencies the second layer of dielectrics becomes a MM so that the “zero averaged refractive index” gaps can be seen in the band structure of PC in both TE and TM polarizations.

According to the effective medium theory, the added atoms will change the electric permittivity of second layer as²¹:

$$\varepsilon'_2 = \varepsilon_2 + (3f_2\varepsilon_2) \frac{1 + \chi_1 - \varepsilon_2}{1 + \chi_1 - 2\varepsilon_2 - f_2(1 + \chi_1 - \varepsilon_2)}, \quad (9)$$

where ε'_2 is the effective electric permittivity of the dielectrics second layer considering the effect of added atoms. f_2 is the filling rate of total volume with three level atoms and χ_1 is the linear susceptibility of the three level atoms that can be defined through the following equation³⁶:

$$\chi^{(1)} = -\frac{\Delta_p}{\Omega_c^2 + i(2 + i\Delta_p)\Delta_p}. \quad (10)$$

In Fig. 1, $\Delta_p = \omega_{13} - \omega_p$ shows the frequency detuning between probe field frequency, ω_p , and the atomic transition resonance frequency ω_{13} . Ω_c and Ω_p are the Rabi frequency of pump and probe fields, respectively.

The optical characteristics of PC are the quantities as given in Eq. (8), except the quantity of ε_2 which changes according to Eq. (9). In Figs. 2(a) and 2(b), we plotted the μ_2 and ε_2 as a function of the radiation field frequency, respectively. Figures 3(a) and 3(b) show the plot of ε'_2 as a function of the radiation field frequency for $f_2 = 0.5$ and $f_2 = 1$, respectively.

Now we calculate the band structure of PC using the optical characteristics of crystal and Eqs. (4) and (5) for different values of f_2 in both TE and TM polarizations. The band structure of PC have been shown in Figs. 4(a), 4(b), 5(a), 5(b), 6(a), 6(b), 7(a), 7(b), 8(a) and 8(b) for $f_2 = 0, 0.3, 0.5, 0.7$ and 1, respectively. In these figures, labels (a) and (b) denotes TE and TM polarization of incident light and red areas indicate the bandgaps of PC in which its transmission is zero.

It is more clear that when the sign of $\mu(\omega)$ or $\varepsilon(\omega)$ changes at the special values of frequency, the gaps related to the “zero averaged refractive index” are found for both TE and TM polarizations. A gap occurs in the band structure of PCs related to TE polarization at the frequencies in which $\mu = 0$ and a gap occurs in the band structure of PCs related to TM polarization at the frequencies in which $\varepsilon = 0$.³⁷⁻⁴⁰

The initial frequencies of bandgaps related to $\mu = 0$ and $\varepsilon = 0$ in Fig. 4 are compatible with diagrams of ε_2 and μ_2 in Fig. 2.

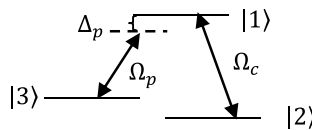


Fig. 1. Diagram of three level atoms.

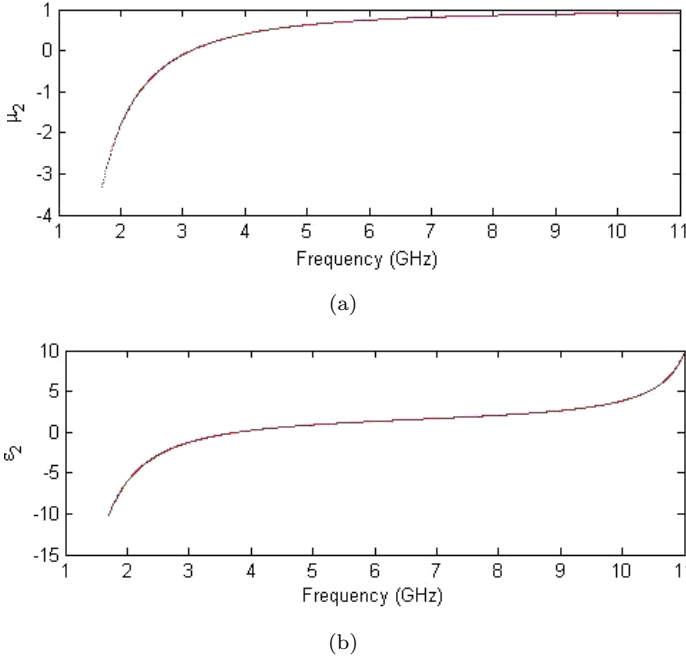


Fig. 2. Diagram in terms of frequency for: (a) μ_2 and (b) ϵ_2 .

The lowest branch of bandgaps in Figs. 4(a) and 4(b) is related to “zero averaged refractive index” in both TE and TM polarizations for which the width of the gap decreases as the angle increases in TM polarization. This behavior did not occur in the TE polarization case where the width of the gap remains constant with angle variation.

The branch above the lowest gap relates to the “zero permittivity” in TM polarization. In addition, the width of this branch increases as the angle increases. However, in TE polarization, this gap does not appear. On the other hand in TE polarization, there is a bandgap related to “zero permeability” in which width increases as the angle increases. The uppermost gaps are related to the “Bragg” bandgap in both polarizations. The gap width at first decreases and then increases as the angle increased in the TM polarization. In TE polarization, by increasing the angle, we only observe an increased mode.

According to Fig. 3(a), ϵ'_2 becomes zero at two points around 4 GHz frequency for $f_2 = 0.5$. We expected to have two gap branches related to “zero permittivity” at these frequencies in TM polarization which is compatible with Fig. 6(b).

In Fig. 3(b), we observed that ϵ'_2 become zero only at one point around the 5 GHz. It means that at $f_2 = 1$, we have only one gap branch related to “zero permittivity” at this frequency in TM polarization which is compatible with Fig. 8(b).

Comparison of Figs. 5(a)–8(b) with Figs. 4(a) and 4(b) convinced us that the PC band structure changes both in TE and TM polarizations as f_2 increases. For

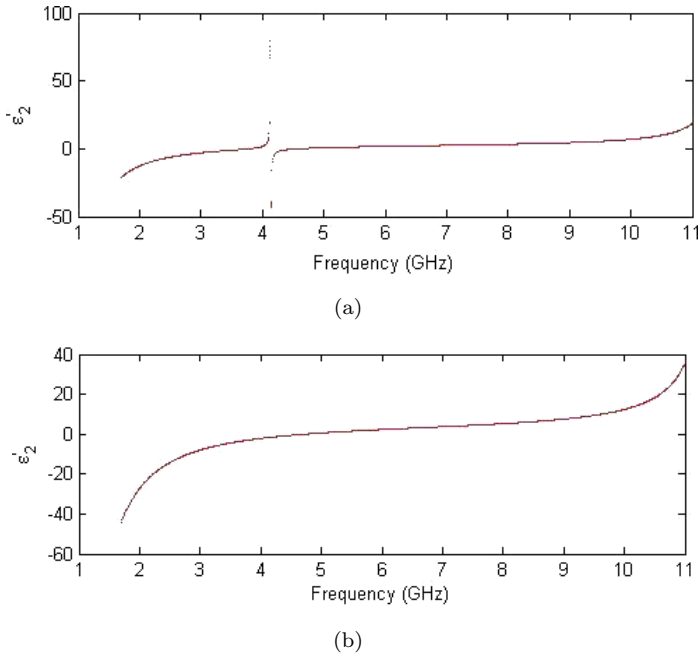


Fig. 3. Diagram of ε'_2 in terms of frequency for: (a) $f_2 = 0.5$ and (b) $f_2 = 1$.

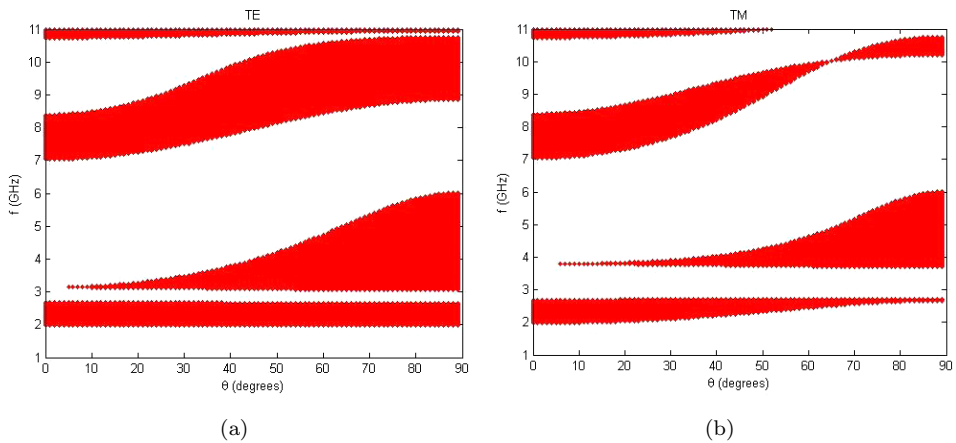


Fig. 4. The PC band structure at $f_2 = 0$ for: (a) TE polarization and (b) TM polarization.

TE polarization, we observed that the bandgap related to “zero averaged refractive index” is gradually broadened. By increasing f_2 , “Bragg” bandgap has also been broadened and slightly shifted toward lower frequencies. Bandgap related to “zero permeability” is narrowed. At 4 GHz, a new gap appears whose width increases as the angle increases. These gaps are gradually broadened and connected to the lower gap. Moreover, a new bandgap appears around the 10 GHz. This gap gradually

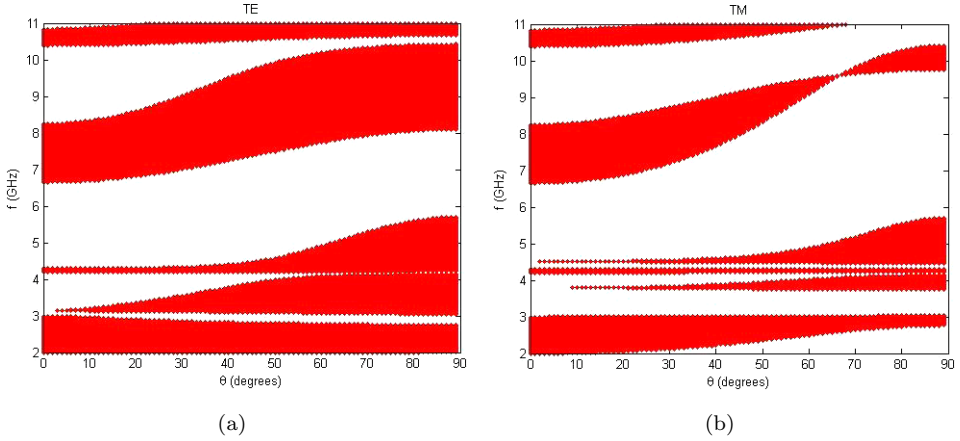


Fig. 5. The PC band structure at $f_2 = 0.3$ for: (a) TE polarization and (b) TM polarization.

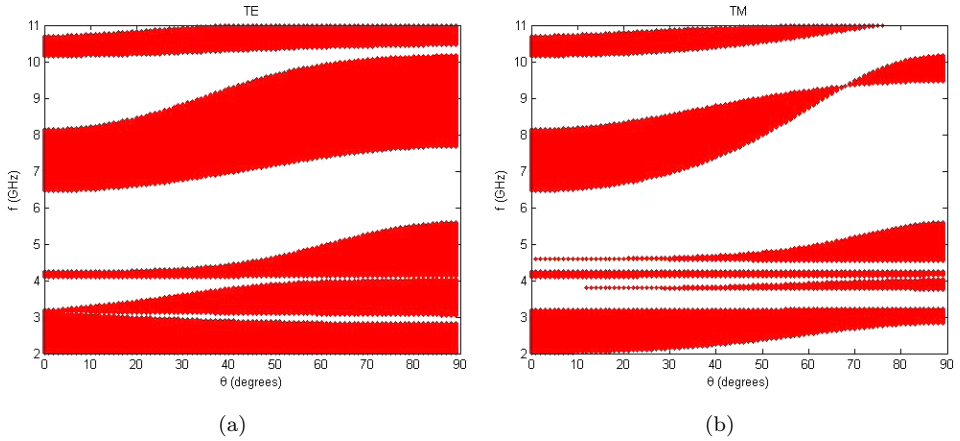


Fig. 6. The PC band structure at $f_2 = 0.5$ for: (a) TE polarization and (b) TM polarization.

broadens by increasing f_2 and finally gets appearance similar to that of below “Bragg” bandgap.

For TM polarization, we find out a similar behavior of TE polarization. Both the “zero averaged refractive index” and “Bragg” gaps broadened and the “Bragg” gap has slightly shifted toward the lower frequencies. A new bandgap appears around the 10 GHz which broadens as f_2 increases and becomes similar to that of previous behavior of “Bragg” gap. Instead of the “zero permittivity” gap, two new gap branches related to “zero permittivity” appears around the 4 GHz which are narrower than that of the previous gap. An “omnidirectional” gap also appears between the “zero permittivity” gaps in which by increasing the angle its width remains constant. By increasing f_2 , the upper “zero permittivity” gap narrowed and slightly shifted toward 5 GHz. But the lower “zero permittivity” and the “omnidirectional” gaps were connected to the “zero averaged refractive index” gap. Such

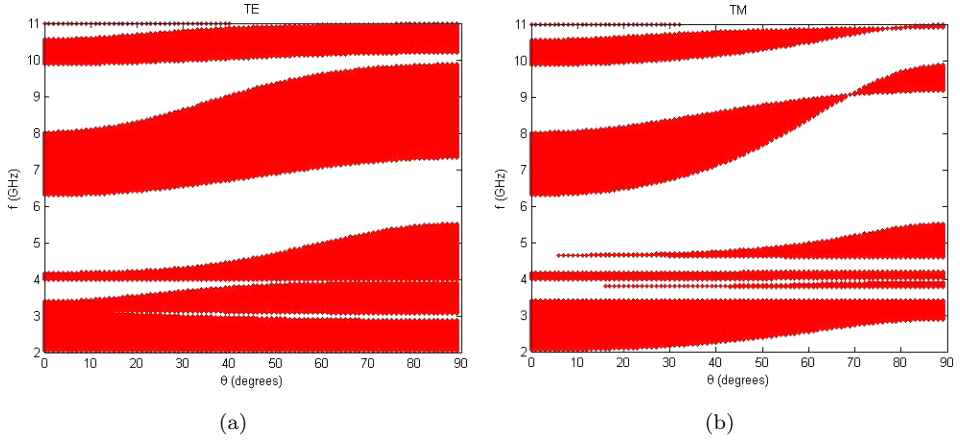


Fig. 7. The PC band structure at $f_2 = 0.7$ for: (a) TE polarization and (b) TM polarization.

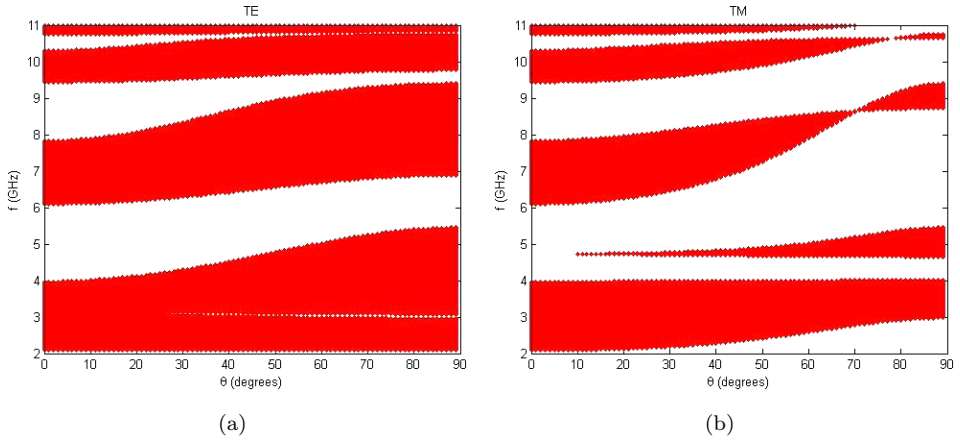


Fig. 8. The PC band structure at $f_2 = 1$ for: (a) TE polarization and (b) TM polarization.

results were physically expected. This is because the PC band structure depends on the electric permittivity and magnetic permeability of the dielectric layers.²¹ So it is expected that adding three level atoms in the dielectrics second layer, according to Eq. (9), causes some fundamental changes in the electric permittivity of this layer and therefore the PC band structure also changes.

4. Conclusion

In brief, in this paper we have calculated the band structure of a 1DPC composed of double-layered dielectrics with specific optical characteristics considering the three level atoms added to the second layer of each dielectric. We have evaluated the effect of these added atoms on the PC band structure for different values of f_2 . Added atoms cause changes in the electric permittivity of second layer. Therefore, the PC

band structure also changes. We observed that by increasing f_2 , the “Bragg” and “zero averaged refractive index” gaps have broadened in both TE and TM polarizations. The “Bragg” gap branches get slightly shifted toward lower frequencies. New bandgaps appeared around 10 GHz in both polarizations, and by increasing f_2 they gradually broadened and became similar to that of lower “Bragg” gap. In TE polarization, “zero permeability” gap has narrowed and a new gap branch appeared at 4 GHz. These gaps have gradually broadened and connected to lower gap. In TM polarization, instead of the previous “zero permittivity” gap, two gap branches related to “zero permittivity” and an “omnidirectional” gap appeared around the 4 GHz. The “omnidirectional” and lower “zero permittivity” gaps stick to “zero averaged refractive index” gap, but the upper “zero permittivity” gap has narrowed and shifted toward higher frequencies.

Acknowledgment

This study was supported by Urmia University in Urmia, Iran (Grant No. 10.195).

References

1. D. Shir *et al.*, *Appl. Phys. Lett.* **94**, 011101 (2009).
2. M. Araghchini *et al.*, *J. Vac. Sci. Technol. B* **29**, 061402-1 (2011).
3. C. C. Cheng and A. Scherer, *J. Vac. Sci. Technol. B* **13**, 2696 (1995).
4. P. R. Villeneuve *et al.*, *IEEE Proc. Optoelec.* **145**, 384 (1998).
5. M. E. Walsh, PhD Thesis, University of MIT, New York, 2004.
6. L. C. Liang, W. Tao and P. J. Xiong, *Optoelectron. Lett.* **6**, 0363 (2010).
7. M. Upadhyay *et al.*, *Indian J. Phys.* doi:10.1007/s12648-015-0760-2 (2015).
8. A. H. AL-Janabi, H. J. Taher and S. M. Laftah, *Indian J. Phys.* **85**, 1299 (2011).
9. H. Hojo, K. Akimoto and A. Mase, *Conf. Digest on 28th Int. Conf. Infrared and Millimeter Waves* (Otsu, Japan, September 28–October 2, 2003), p. 347.
10. H. Hojo *et al.*, *Plasma Fusion Res.* **1**, 021-1-2 (2006).
11. G. Guida, A. de-Lustrac and A. Priou, *PIER* **41**, 1 (2003).
12. E. Yablonovitch, *Phys. Rev. Lett.* **58**, 2059 (1987).
13. S. John, *Phys. Rev. Lett.* **58**, 2486 (1987).
14. L. G. Wang, H. Chen and S. Y. Zhu, *Phys. Rev. B* **70**, 245102 (2004).
15. R. A. Depine *et al.*, *Phys. Lett. A* **364**, 352 (2007).
16. Z. Wang and D. Liu, *Appl. Phys. B* **86**, 473 (2007).
17. B. Guo and X. M. Qiu, *Optik* **123**, 1390 (2012).
18. Z. Y. Li and L. L. Lin, *Phys. Rev. E* **67**, 046607 (2003).
19. J. J. Wu and J. X. Gao, *J. Supercond. Nov. Magn.* **27**, 667 (2014).
20. A. Jafari and A. Rahmat, *Indian J. Phys.*, doi:10.1007/s12648-016-0934-6 (2016).
21. A. H. Sihvola, *Electromagnetic Mixing Formulas and Applications*, eds. P. J. B. Claricoats *et al.* (Institution of Electrical Engineers, United Kingdom, 1999), p. 48.
22. I. A. Sukhoivanov and I. V. Guryev, in *Photonic Crystals: Physics and Practical Modeling*, ed. W. T. Rhodes (Heidelberg, Springer, 2009), p. 27.
23. A. Yariv and P. Yeh, *Optical Waves in Crystals*, ed. P. Yeh (John Wiley & Sons, New York, 1984), p. 155.
24. R. W. Boyd, *Nonlinear Optics*, eds. G. P. A *et al.* (Academic Press, San Diego, 2003), p. 194.

25. P. Yeh, A. Yariv and C. S. Hong, *Opt. Soc. Am.* **67**, 423 (1977).
26. P. Yeh, *Optical Waves in Layered Media*, ed. B. E. A. Saleh (John Wiley & Sons, New York, 1998), p. 102.
27. J. B. Pendry and A. Mackinnon, *Phys. Rev. Lett.* **69**, 2772 (1992).
28. N. H. Liu *et al.*, *Phys. Rev. E* **65**, 046607 (2002).
29. W. L. Gang *et al.*, *Phys. Rev. E* **70**, 016601 (2004).
30. V. G. Veselago, *Appl. Phys. B* **81**, 403 (2005).
31. L. Qi *et al.*, *Phys. Plasmas* **17**, 042501-1-8 (2010).
32. M. Born and E. Wolf, *Principles of Optics* (Cambridge University Press, Cambridge, 1999), p. 38.
33. J. N. Winn *et al.*, *Opt. Lett.* **23**, 1573 (1998).
34. Y. Fink *et al.*, *Science* **282**, 1679 (1998).
35. M. Mansuripur, *Opt. Photon. News* **9**, 8 (1998).
36. S. Aas and O. E. Müstecaplıoğlu, *Phys. Rev. A* **88**, 053846 (2013).
37. J. Li *et al.*, *Phys. Rev. Lett.* **90**, 083901 (2003).
38. H. Jiang *et al.*, *Phys. Rev. E* **69**, 066607 (2004).
39. N. Garcia, E. Ponzovskaya and J. Xiao, *Appl. Phys. Lett.* **80**, 1120 (2002).
40. B. Schwartz and R. Piestun, *J. Opt. Soc. Am. B* **20**, 2448 (2003).