

# Collapse Behavior of Single-Layer Space Barrel Vaults under Non-Uniform Support Settlements

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## Abstract

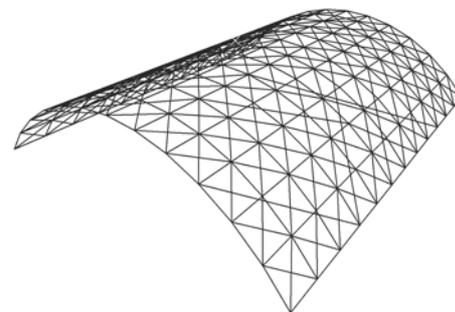
Single-layer space barrel vaults are appropriate structures for covering extensive spaces with large spans for which non-uniform support settlement is considered as a great concern. This can affect the stability and collapse behavior of these structures. Accordingly, in this paper, the collapse behavior of the barrel vaults with different slope angles is investigated under various types of the non-uniform support settlement. Both material and geometrical nonlinearity are taken into account while studying the effects of the settlements on the collapse progression in barrel vaults. Finally, the optimum slope angles of the structures are determined so that it minimizes the destructive effects of the support settlement. Moreover, the allowable values of the non-uniform support settlements provided by several code provisions have been investigated in detail.

**Keywords:** support settlement, single-layer space structure, barrel vault, nonlinear analyze, collapse behavior

## 1. Introduction

The latticed space structures are a group of three-dimensional structural systems that consist of straight bar or beam elements. These structures possess some noteworthy features such as high stiffness, light structural weight, and elegant architecture. These advantages all are due to the well-distribution nature of their internal forces. Moreover, Industrial developments in construction, easy installation techniques, variability in forms, and their architectural elegance are some other sensible reasons which justify their ever-increasing developments in recent decades. Now, they are mostly used to cover large spaces in which intermediate supports are not desired.

Barrel-vaults are also a special family of latticed space structures which are arched in one direction. The curve shape of their sections can be a cut of a circle, ellipse, or cycloid. The barrel vaults are widely used for covering railway stations, hangars, industrial and sports salons and can be manufactured in single-layer, double-layer, or multi-layer configurations. According to stability considerations,



**Figure 1.** A single layer barrel vault.

the single-layer type is constructed with rigid connections (Fig. 1).

All structures, especially those ones which have large spans, may undergo support settlements during their serviceability lifetime. Now, if the settlement values are not the same in all supports, the settlement of structure will be called non-uniform settlement. This kind of settlement can affect the behavior of the structure by creating additional internal forces and stresses. Some determining factors such as ground profile variances, difference in the loads acting on the various parts of the structure, fabrication errors, and the application of various foundation types are the main reasons of the non-uniform support settlements. The predominant role of these factors and the appearance of the non-uniform support settlement can be more determining in design of

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the space structures rather than any other ones due to their large span length. The forces arisen from non-uniform support settlement may have destructive effects on the structural response of these systems containing slender elements that are susceptible to buckling phenomenon. This in turn affects the collapse behavior of the structure.

Several studies have been done on the effect of non-uniform support settlement on the different kinds of structures. Zhang and Tan (2010) studied the effect of non-uniform support settlement on the strength and behavior of continuous deep beams with numerical and experimental methods. Cao and Zhao (2010) investigated the buckling strength of large vertical cylindrical steel storage tankers under harmonic settlements. Godoy and Sosa (2003) studied the effect of this phenomenon on out-of-plane displacements of thin-walled storage tanks. They assumed the settlement variation in the supports experiencing non-uniform settlement to be linear. Gilbert and Sakka (2010) studied the strength and ductility of one-way reinforced concrete slabs subjected to support settlement. They concluded that the differential support settlement has a significant effect on the behavior and strength of one-way reinforced concrete continuous slabs. Atamturktur *et al.* (2011) studied the vibration specifications of arched stone buildings undergoing non-uniform support settlement.

Relating to the space structures, Shugar and Holland (1990) studied the collapse of a latticed dome due to the support settlement. El-sheikh (1996) also investigated the sensitivity of space trusses to uneven support settlement. In this work, the collapse load of various models with different rise-to-span ratios are assessed under different conditions of support settlements. In addition, the value of the settlements in settled supports are assumed to be equal.

Gioncu (1995) claimed that reticulated shells show

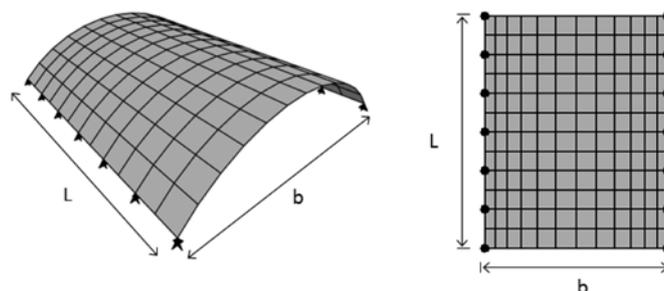
significant nonlinearity with considerable softening behavior. And in single layer reticulated shells, the role of geometrical nonlinearity becomes more dominant while the material nonlinearity takes less importance. Changes in the member lengths due to axial and bending forces, rotation in rigidly connected joints, and manufacturing errors in connection systems are the main sources of geometrical nonlinearities in these structures.

In this study, the collapse behavior of single layer space barrel vaults under different cases of non-uniform support settlements have been investigated through non-linear analysis, and the effect of these settlements on the collapse load have been studied in detail. Also, based on the obtained results, the validity of the allowable settlement values presented in different references have been assessed. Since geometry characteristics are more effective factors on the behavior of structures, the ratio of rise-to-span have been considered to be investigated here as an efficient variance in the study of the collapse behavior of the barrel vaults.

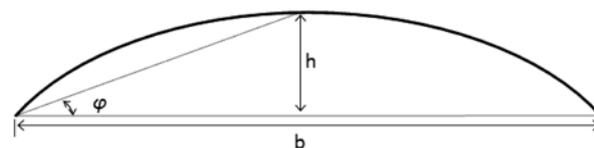
## 2. Morphology of the Single-layer Barrel Vaults

The results of the several studies show that the rise-to-span ratio is an important factor on load bearing capacity of vault structures. Mohammadi *et al.* (2012) investigated the stability of single layer Barrel Vaults. They concluded that variations of rise-to-span ratios have significant effect on the load bearing capacity of vaults and the critical ratio is equal to 0.2. Accordingly, in this paper, in order to study the effects of the non-uniform support settlement on the collapse behavior of the single-layer barrel vaults, a survey is conducted on four models with different rise-to-span ratios.

Each model is a curved grid with  $12 \times 12$  square meshes



(a) The configuration and boundary conditions



(b) The slope angle of the barrel vault

**Figure 2.** The geometry properties.

**Table 1.** The numerical values of geometry properties

Model	$\phi$ (°)	h (m)	b (m)	L (m)
B1	15	2.59	22	23.04
B2	20	4.00	22	23.76
B3	25	5.13	22	24.96
B4	30	6.35	22	26.52

and their sections are part of an arch cut from a circle. Geometric configurations of the Barrel Vaults are created using the formex algebra configuration processing that is extended by Nooshin and Disney (2000).

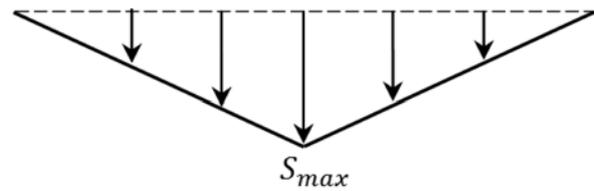
Barrel Vaults can be supported along either their longitudinal edges or transverse edges or both of them. However, the position of the supports plays an important role in the structural behavior and load bearing capacity of the Barrel Vaults (El-Sheikh, 2002). In this study, longitudinal supporting system is used for each model. Geometrical characteristics, configuration, and boundary conditions of the barrel vaults under investigation have been shown in Fig. 2. The slope angle,  $\phi$ , is defined as the ratio of the rise, h, to the half-span, (b/2). The numerical values of the geometry characteristics for each of the models have been presented in Table 1.

### 3. Structural Design of the Models

As shown in Fig. 2, the barrel vaults supported along their longitudinal edges, known as arch barrel vaults (El-sheikh, 2002), are considered to be investigated in this study. In these types of the barrel vaults, transverse members have the task of transferring the applied loads to the supports. Both dead and snow loads are involved in designing process. Dead loads from structural weight, cladding and connections have been separately calculated for each model. The snow load has been considered  $1500 \text{ N/m}^2$  which will be distributed among the nodes according to the slope of the surface. The models have been designed after performing a linear static analysis and determining the initial forces of the members. In the analysis and design, the module of elasticity and the yielding stress have been considered to be  $210 \text{ kN/mm}^2$  and  $240 \text{ N/mm}^2$ , respectively. Two different types of cross section have been used for designing the members. The sections all are box-sections as presented in Table 2. Due to the flexural functioning of the members in single layer barrel vaults, these sections would present more appropriate behavior in bending.

**Table 2.** Member sections of the barrel vaults

Model	Type 1	Type 2
B1	12×12×5SHS	14×14×5SHS
B2	12×12×5SHS	14×14×5SHS
B3	12×12×5SHS	14×14×5.6SHS
B4	12×12×5SHS	14×14×7.1SHS

**Figure 3.** Linear settlement model.**Table 3.** Tolerable differential settlement of buildings (mm)

Criterion	Isolated foundations	Rafts
Greatest differential settlement		
Clays	45	
Sands	32	
Maximum settlement		
Clays	75	75-125
Sands	50	50-75

### 4. Non-Uniform Support Settlement

The response of the structures under settlement in one or a group of supports has been investigated in this study. The model of linearly changed settlement is employed to simulate the settlements occurred in the adjacent supports, which it may include three or five ones. In this model, it is assumed that the maximum settlement takes place in the mid-length support of the settled region (Fig. 3).

To investigate the effects of the non-uniform support settlement on the collapse behavior, each of the settlement cases have been applied to the structures with different conditions and values. The recommended values of settlements are taken in the range of those ones presented in the available codes. On the other hand, different references present dissimilar values for the allowable settlements in structures. According to USSR building code (Bowles, 1997), the maximum value of the settlement in steel frames is limited to 100 mm. Table 3 shows the recommended values for differential and maximum settlements in steel and reinforced concrete constructions (Bowles, 1997).

The numbering of the settled supports has been shown in Fig. 4. and different cases of non-uniform support settlement have been presented in Tables 4 to 6. In these tables S1, S2, ..., and S6 are settlement values of supports 1, 2, ..., and 6, respectively. They have been determined based on the linear model of settlements.

### 5. Structural Analysis Methodology

The more accurate investigation into the effects of the non-uniform support settlement on the structural response would be possible by conducting non-linear finite element analyses. For this purpose, an incremental non-linear static analysis has been carried out. Both the geometrical

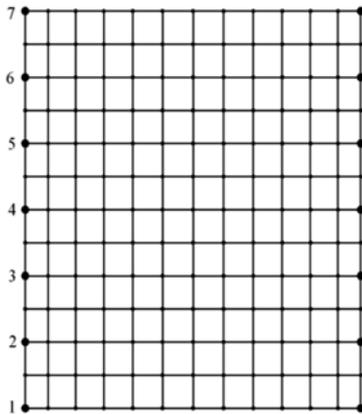


Figure 4. The numbering of settled supports.

Table 4. The settlement values for the case of the settlement in one support (cm)

Settlement type	S1	S4
S111*	2.5	-
S112	5	-
S113	7.5	-
S141	-	2.5
S142	-	5
S143	-	7.5

\*The first digit is the number of settled supports, the second is the middle one, and the third represent the case corresponding to the value of settlement

Table 5. The settlement values for the case of the settlement in three supports (cm)

Settlement type	S1	S2	S3	S4	S5
S321	2.5	5	2.5	-	-
S322	5	10	5	-	-
S323	7.5	15	7.5	-	-
S331	-	2.5	5	2.5	-
S332	-	5	10	5	-
S333	-	7.5	15	7.5	-
S341	-	-	2.5	5	2.5
S342	-	-	5	10	5
S343	-	-	7.5	15	7.5

Table 6. The settlement values for the case of the settlement in five supports (cm)

Settlement type	S1	S2	S3	S4	S5	S6
S531	2.5	5	7.5	5	2.5	-
S532	5	10	15	10	5	-
S533	7.5	15	22.5	15	7.5	-
S541	-	2.5	5	7.5	5	2.5
S542	-	5	10	15	10	5
S543	-	7.5	15	22.5	15	7.5

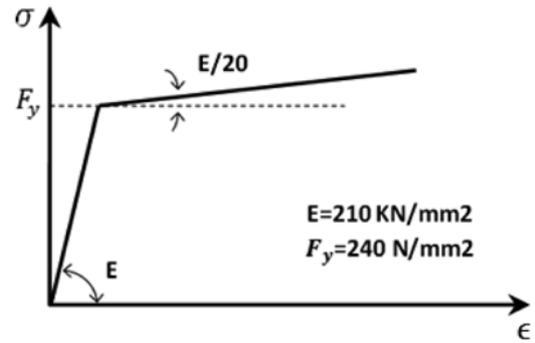


Figure 5. Material properties assumed for members.

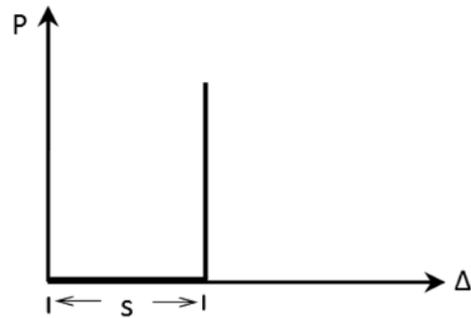
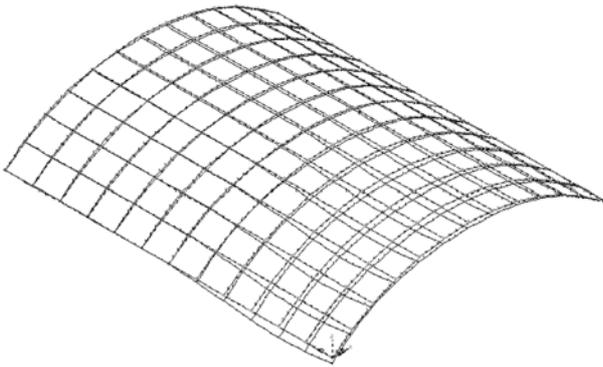


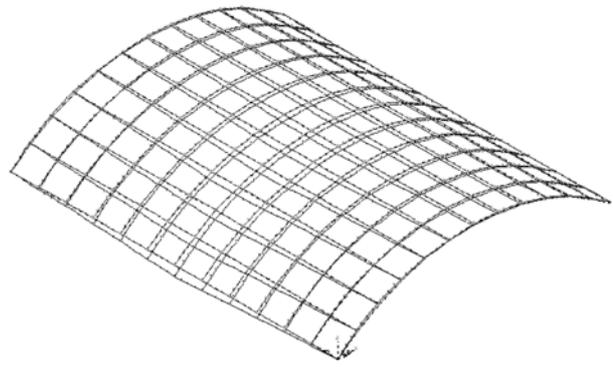
Figure 6. Behavior of under settlement support that modeled with joint element.

and material nonlinearities have been considered in the analysis. The idealized stress-strain diagram of the steel material is given in Fig. 5. In order to apply geometrical non-linearity effects, the Total Lagrangian approach has been employed. This method is an appropriate approach for large rotations and it can be used for elastoplastic problems (Crisfield, 2000).

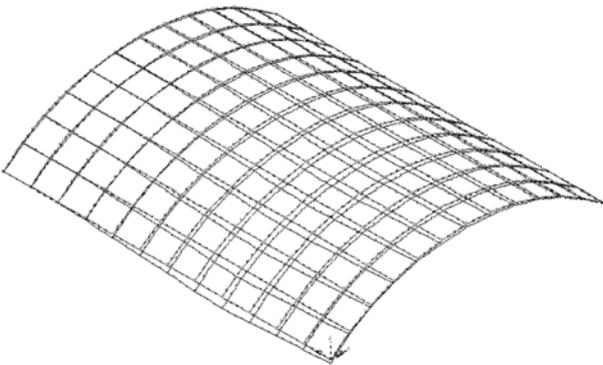
The stability of the single layer latticed structures can be guaranteed using rigid joints. Therefore, welded connection system is applied to the single layer barrel vaults. Consequently, in the structures with rigid joints, structural elements show flexible behavior. Then, Kirchhoff beam element with 6 degrees of freedom per joint has been used for the finite element model. To model the support settlement, a *JOINT* type element, which can be assigned a pre-specified gap, was employed in this model. So, supports have been modeled with a pre-specified gap space, and then the structures are analyzed using an incremental non-linear static method. While the loads increase incrementally, structure settles freely on the supporting bed until the gap fills. Finally, when the gaps are completely closed, the constraints prevent the structure from extra settlement. Figure 6 shows the behavior of a settled support that is modeled using the aforementioned model. To solve the obtained nonlinear equilibrium equations, the robust technique of Crisfield method is implemented which is one of the arch length methods.



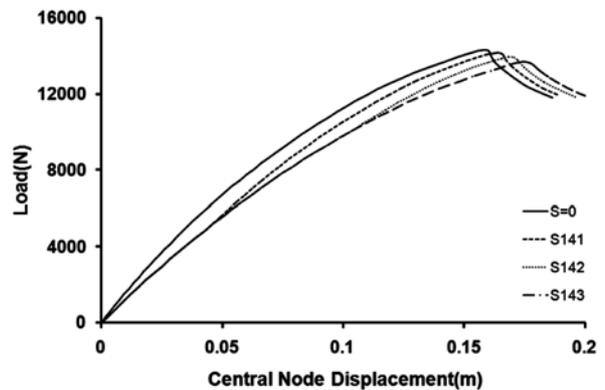
**Figure 7.** The undeformed and deformed shape of the B2 barrel vault under S323 settlement.



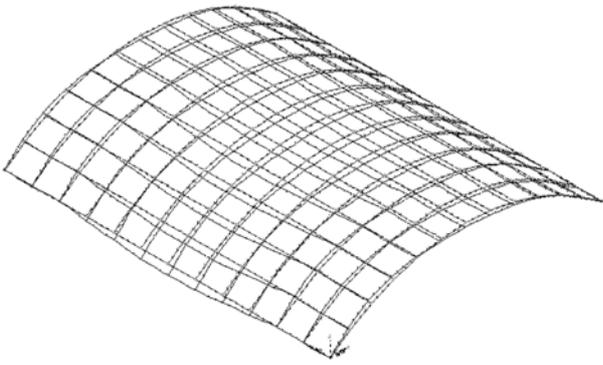
**Figure 10.** The undeformed and deformed shape of the B2 barrel vault under S543 settlement.



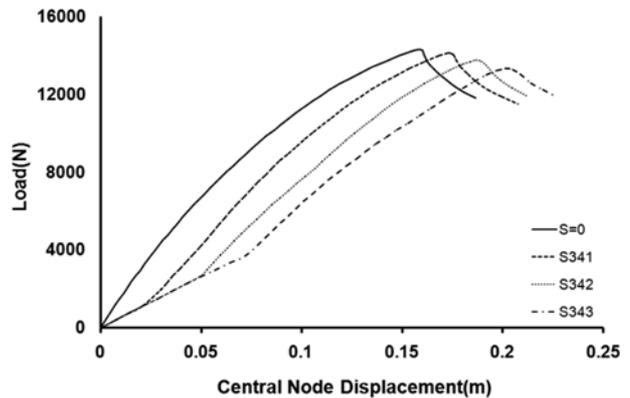
**Figure 8.** The undeformed and deformed shape of the B2 barrel vault under S533 settlement.



**Figure 11.** The load-displacement response of the barrel vault B2 under the S141, S142 and S143 settlements.



**Figure 9.** The undeformed and deformed shape of the B2 barrel vault under S343 settlement.



**Figure 12.** The load-displacement response of the barrel vault B2 under the S341, S342 and S343 settlements.

## 6. Analysis Results

Results obtained from non-linear analysis suggest that in single layer space barrel vaults with edge supports, transverse members attached to the settled supports were the most critical ones, and collapse starts from these elements and extends toward the central elements of the barrel vault while the load increases. The undeformed and deformed shape of the B2 barrel vault under S323, S533, S343 and S543 settlements have been shown in Fig. 7-10.

The load-displacement response of the middle node in the models B2 and B4 possessing the slope angle of  $20^\circ$  and  $30^\circ$ , respectively, is shown in Fig. 11-16. Precisely analyzing these diagrams, we can conclude that the support settlement leads to a reduction in collapse load of these structures. The more the settlement value is, the more reduction in this capacity appears. In each model discussed above, the structure provides smaller stiffness during appearance the settlement compared with the model

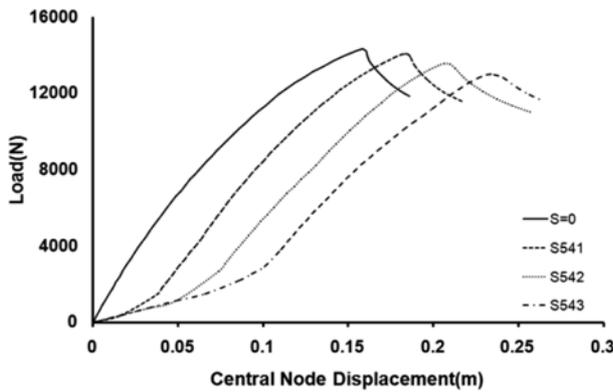


Figure 13. The load-displacement response of the barrel vault B2 under the S541, S542 and S543 settlements.

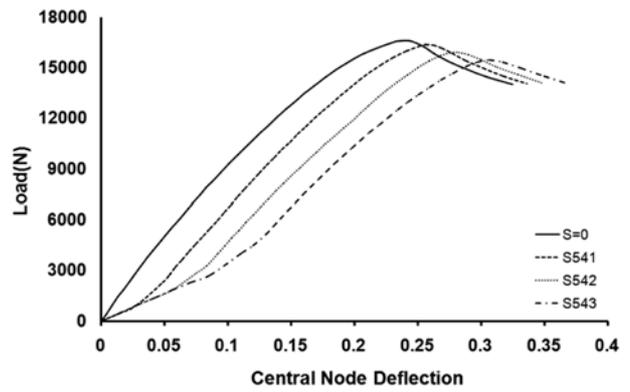


Figure 16. The load-displacement response of the barrel vault B4 under the S541, S542 and S543 settlements.

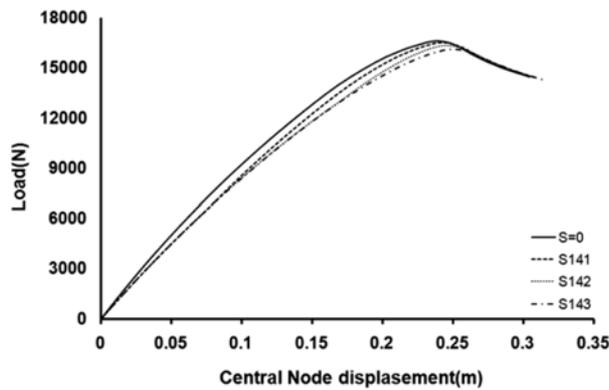


Figure 14. The load-displacement response of the barrel vault B4 under the S141, S142 and S143 settlements.

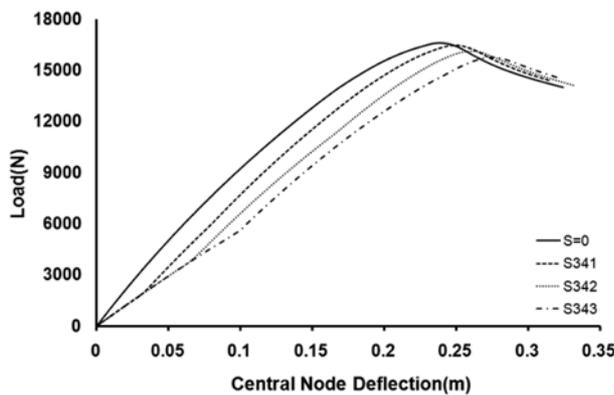


Figure 15. The load-displacement response of the barrel vault B4 under the S341, S342 and S343 settlements.

without settlement, but after completion of the settlement at the considered supports, its behavior inclines toward the behavior of the original structure without settlement.

The loss of collapse load in each model under different types of settlements has been shown in Tables 7-9. According to the results, the maximum reduction in capacity was 9.42% for model B2 under S543 settlement. Moreover, the results reveal that there is a high degree of

Table 7. The loss of collapse load in each model under different forms of settlement in one support (%)

Settlement type	B1	B2	B3	B4
S111	0.74	1.40	0.95	0.89
S112	1.26	2.64	1.80	1.69
S113	1.78	3.85	2.70	2.59
S141	0.11	0.91	0.40	0.59
S142	0.65	2.42	1.65	1.67
S143	1.41	4.29	3.00	2.86

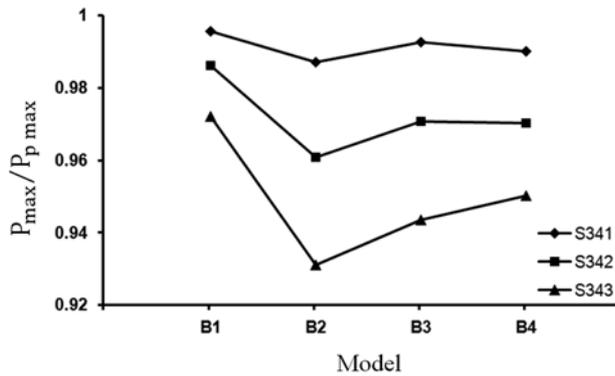
Table 8. The loss of Collapse load in each model under different forms of settlement in three supports (%)

Settlement type	B1	B2	B3	B4
S321	0.87	2.10	1.43	1.45
S322	1.77	4.27	3.13	3.03
S323	2.88	5.54	5.18	4.71
S331	0.39	1.44	0.95	1.09
S332	1.33	4.03	3.04	3.00
S333	2.71	6.87	5.48	5.01
S341	0.43	1.30	0.74	1.01
S342	1.38	3.92	2.93	2.98
S343	2.79	6.89	5.65	4.98

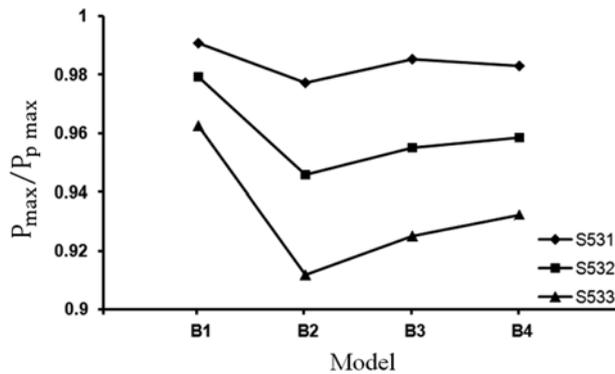
Table 9. The loss of Collapse load in each model under different forms of settlement in five supports (%)

Settlement type	B1	B2	B3	B4
S531	0.91	2.28	1.48	1.70
S532	2.08	5.41	4.50	4.15
S533	3.75	8.83	7.48	6.77
S541	0.39	1.65	1.11	1.44
S542	1.55	5.29	4.28	4.24
S543	3.54	9.42	7.80	6.95

sensitivity toward the settlement in the middle-span supports in comparison with the corner supports.



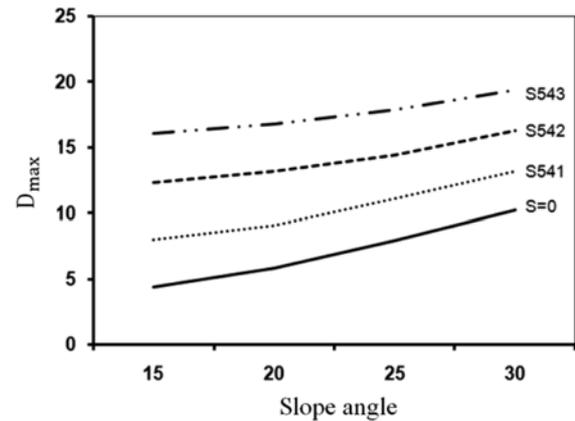
**Figure 17.** The Effect of the slope angle on the collapse load of the Barrel vaults under S341, S342 and s343 settlements.



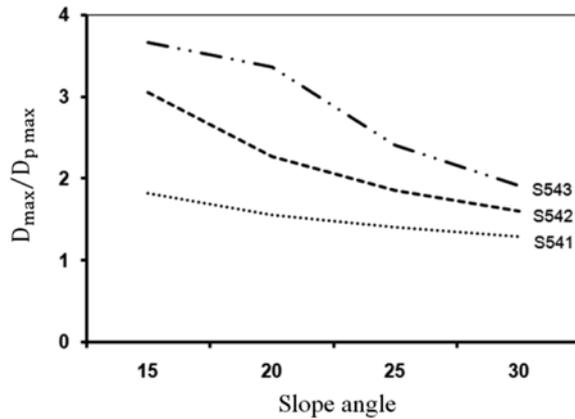
**Figure 18.** The Effect of the slope angle on the collapse load of the Barrel vaults under S531, S532 and S533 settlements.

## 7. Effects of the Slope Angel on Collapse Load

The results from the previous section imply that the slop angle is an important and affective factor in the collapse behavior of the single layer space barrel vaults. The collapse load variations under different case of support settlements versus to the slop angle of the barrel-vaults is shown in Figs. 17-18. In these figures the vertical axes represent the ratio of the collapse load of the settled barrel vault to the collapse load of the perfect structure. As it can be seen the model B2, with the slop angle of  $20^\circ$ , shows high sensitivity against the non-uniform settlements, and increase in the collapse load of this model is greater than those of which are observed in the other models with similar settlements. Moreover, increase in collapse load of model B1, which has a slope angle of  $15^\circ$ , is less than the others. In other words, this model presents minimum sensitivity toward support settlement among the other tested models.



**Figure 19.** The Effect of the slope angle on the maximum deflection of the Barrel vaults under S541, S542 and S543 settlements.



**Figure 20.** The Ratio of maximum deflection in the structure under settlement to the maximum deflection of the structure without settlement for B2 barrel vault.

## 8. Effects of the Slope Angle on Deflection

Analysis results of the models indicate that the maximum vertical deflection in structures under settlement is greater compared with the structures without settlement. Figure 19 shows the maximum vertical deflection varies with respect to the changes in the barrel vault slop angle for similar settlements. It is clear from this figure that the maximum vertical deflection increases as the barrel vault slope increases.

The ratio of the maximum deflection, arisen in the structures under settlement, decreases up to the maximum deflection in the structures without settlement (Fig. 20). It is obvious that in the structures under settlement, in addition to the load bearing criterion, maximum deflection is also important for appropriate serviceability conditions. According to Iranian Code of Practice for Skeletal Steel

Space Structures (2010), the maximum allowable deflection for the space structures under gravity loads is 1/150 of span length. Then, the allowable deflection will be 16.6 cm for the models according to their span length of 22 m. Investigation in the maximum deflection of the models under settlement shows that in the model B2 the case of S543 settlement; in model B3, the cases of S533 and S543 settlements; and in model B4, the cases of S532, S533, S542 and S543 the settlements lead to a greater deflections which exceed the allowable values from the regulations.

## 9. Conclusion

The results obtained from the investigation into the collapse behavior of the single layer space barrel vaults under non-uniform support settlements in the scope of the studied models are as follows:

(1) Changing the slope angle of the single layer space barrel vaults affects the collapse load of the single layer space barrel vaults in a way that the slope angle of 20° has presented the maximum sensitivity toward the non-uniform support settlements. Moreover, minimum loss in load-bearing capacity is occurred with a slope angle of 15°.

(2) In the barrel vaults undergoing support settlements, collapse initiates from that parts of the structure which is close to the supports experiencing settlement. Then the collapse progresses similar to the structures without settlement.

(3) The positions of the supports experiencing settlement have a direct effect on the collapse behavior and load-bearing capacity of the single layer space barrel vaults. Accordingly, the settlement taken place in the middle supports highly decreases the collapse load of the structures.

(4) The comparison between the allowable settlement values provided in the references and that one obtained in this study indicates that the single-layer space barrel vaults has the ability of bearing the greater value of the settlements than those of which come in the references.

(5) The slope angle is also an effective factor in the deflection of the barrel vaults. By increasing the value of this angle, deflection also increases in the similar case of the settlements.

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