Effect of Random Distribution of Member Length Imperfection on Collapse Behavior and Reliability of Flat Double-Layer Grid Space Structures

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Abstract: The existence of initial imperfections in manufacturing of double-layer space structures having thousands of members is inevitable. Many of the imperfections, such as member length imperfections, are all random in nature. In this paper, the probabilistic effect of member length imperfection in the load bearing capacity of double-layer grid space structures with different types of supports have been investigated. First, for the member length imperfection of each member, a random number is generated from a normal distribution. Then, the imperfections are randomly distributed amongst the members of the structure. Afterwards, the ultimate bearing capacity of the structure is determined by using nonlinear analysis and this procedure is frequently repeated using Monte Carlo simulation method. Ultimately, based on the maximum values of bearing capacity, structure's reliability diagrams are obtained. The results show the sensitivity of the collapse behav ior of double-layer grid space structures to the random distribution of initial imperfections.

Key words: double-layer grids, random initial imperfections, probability, reliability, Monte Carlo simulation method, progressive collapse.

1. INTRODUCTION

In recent years, for advantages of space structure such as slight weight and high load bearing capacity, these structures are used for covering large areas such as sport and multi-purpose saloons. Space trusses of double-layer grids usually have high static indeterminacy. Affan estimated the number of redundant bars that could be removed without affecting truss stability, at 15% to 25% of the total number of truss members. This has led some designers to mistakenly believe that space trusses are highly reliable because the structure would still stand after removing a large number of redundant members (Affan 1987; Affan and Calladine 1989).

Schmidt *et al*. (1980, 1982) studied different types of trusses, he showed that, when a truss member yields, the truss loses stiffness, but however continues to carry more load. On the other hand, compression member buckling proved to be more critical. After buckling, a member loses most of its strength, thus shedding force to neighboring members, and frequently causing them to buckle as well. This would cause spreading member failure and progressive collapse which could take only a few seconds to develop. The number of members that following their buckling truss changes to mechanism in less than a few seconds, is lower than the number reported by Affan (Schmidt *et al*. 1982).

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Schmidt *et al*. (1980) also found, to the surprise of many, that space trusses' high statical indeterminacy not only failed to arrest the progressive nature of truss collapse, but was also partly responsible for developing initial member forces due to lack of fit. These forces could result in the premature buckling of a critical compression member, hence triggering an early structural collapse. Member's lack of fit, the most common form of member geometric imperfections, is known to be of common occurrence in practical trusses. In structures that typically contain hundreds or thousands of members, precise manufacturing throughout is almost impossible to achieve, and the existence of some imperfect members is always inevitable. Furthermore, description of a response of real-life structural systems is inevitably associated with various sources of uncertainties or random variables. Besides, basically the response of these structures itself has stochastic behavior. Therefore, in order to estimate, and furthermore, to insure the safety of a double-layer space structure, it is necessary to consider the effects of uncertainties in system parameters. In this regard, many investigations have been conducted by numbers of researchers, each including one or some of these random variables.

Wada and Wang (1992) studied the influence of random variation of member strength and construction errors on the mechanical behavior of double-layer space structures. They showed that construction errors, like assembly errors have enormous influence on the load carrying capacity of these structures. El-Sheikh (1995) presented a work on the study of the sensitivity of double-layer space structures to member geometric imperfections on overall strength and behavior and the location of truss critical areas at which imperfect members should be avoided. El-sheikh (1997) also has investigated the effect of member length imperfections on capacity and failure mechanism of triple-layer space structures.

El-sheikh (2002) also has investigated the effect of geometric imperfections on capacity and failure mechanism of single-layer barrel vaults, he presented that geometrical imperfections causes a decrease in the load bearing capacity. Zhao *et al*. (2014) investigated the effects of random geometrical imperfections on concentrically braced frames and they showed that these imperfections have a substantial effect on design forces. They calculated the forces in the braces and their probabilistic distribution by using the Monte Carlo simulation method. De Paor *et al*. (2012) investigated the effects of random geometrical imperfections in shell cylinders and based on several experimental and numerical samples, concluded that the random distribution of imperfections has a good concurrence with the normal distribution. Vryzidis *et al*. (2013) investigated the effect of random initial imperfection of steel pipes in their buckling capacity. Numerical analyses and experimental results have shown that not only initial imperfection has noticeable effects on the system's buckling capacity, but it affects the failure mode of the pipes as well. Kala (2013) investigated the effects of random distribution of imperfections on the lateral-torsional buckling of the **I**-shaped hot rolled beams with simple supports. He concluded that the cross-section imperfection causes a decrease in the bearing capacity.

In this paper the effect of length imperfection in the bearing capacity of double-layer grids has been probabilistically investigated. Length imperfections distribution among the members have been considered probabilistically and the structure's reliability for different supports has been studied using the Monte Carlo simulation method. Considering the nonlinear behaviors and the existence of hundreds of random variables, calculating the structural reliability is very costly and time consuming. Therefore, in this paper, the structure's reliability has been obtained through a direct and simpler approach. All of the analyses have been carried out using the finite element software OpenSees (McKenna *et al*. 2010).

2. MONTE CARLO SIMULATION METHOD

In many circumstances, it is impossible to mathematically describe the response of some structural systems because of their random nature and the complexity of the problem. Even when we can find a mathematical model to predict the behavior of the system, there's no closed form solution through which we can solve the equation. In such cases, simulation is one of the most productive techniques to acquire information about complex problems. As a matter of fact, simulation is a special technique to approximate the quantities that are difficult to obtain analytically. Amongst many of simulation procedures, the Monte Carlo simulation method is one of the most well-known and common procedures in solving complex engineering problems. The base of all simulation procedures is to generate numbers that are uniformly distributed between zero to one. These random numbers are generated by using a computer and random number generating algorithms. In the OpenSees software, standardized algorithms in the C++ programming language have been used to generate random numbers (Haukaas 2003).

In the Monte Carlo simulation method, the probabilistic distribution of the inputs will be determined and based on that the input vectors will be generated. Then, a lot of analyses are carried out and the probability of the collapse of the system will be calculated. In reality, the probability of the collapse will be obtained by generating a limited number of random numbers. Therefore, the calculated collapse is only an estimation of the real collapse probability of the system. By increasing the number of simulations, the estimation will get closer to the actual value. Although this method is very costly and time consuming, but with computers and analysis algorithms constantly getting modified and more powerful, it is used in a wide range of engineering problems (Nowak and Collins 2000)

3. ANALYTICAL MODELS

In this paper, a 7 × 7 bay offset double-layer grid structure has been studied. The plan dimensions of the lower layer and the upper layer are 21 × 21 and 18 × 18 square meters respectively and the grid depth has been considered 2 meters. Each structure is composed of 392 members with the length of 2.5 or 3 meters. The supports have been considered in three different positions: Corner supports, Edge supports and surrounding supports. All supports are hinged as shown in Figure 1. All members are assumed to be steel pipes and the material's yield stress, ultimate stress, and elasticity modulus have been considered 240 MPa, 360 MPa and 210000 MPa respectively.

The design of the structure has been done under two different types of loads. The dead load which is equal to 0.5 kN/m^2, consists of the load of the covering and the joints. And the snow load which is equal to 2.0 kN/m^2. In this model, the dead loads and the snow load have been exerted on the joints of the upper layer as concentrated loads, proportionate to the load bearing area of each joint. As an exception, the four diagonal members over the support nodes are considered to be solid bars so that the local collapse of these members and the instability of the entire structure will be prevented.

The structure has been designed using AISC allowable stress design method and to achieve minimum structural weight it was observed that compression is the dominant response of the system. Design is based on optimizations according to the item of the weight, five sections with specified characteristics in Table 1, are used (AISC 2010).

4. THE ANALYSIS PROCEDURE

The existence of length imperfections causes initial deformations and stresses in the structure. To investigate the effects of this type of imperfection in the structure, the nonlinear finite element method can be employed.

To carry out the static nonlinear analysis for assessing the collapse behavior of space structures, first the nonlinear behavior of each member will be determined. In this paper, the axial load-axial displacement behavior of the members in tension has been considered to be elastic perfectly plastic.

To determine the axial load-axial displacement response of compression members, the nonlinear static analysis has been utilized. To obtain the governing buckling mode of the members, an initial curvature with the maximum lateral deflection of 0.001L is considered (Sheidaii and Abedi 2003).

The initial geometrical imperfection has been considered as a sinusoidal half-wave in such a way that the maximum deviance will be in the mid-span of the member (Figure 2). This member was created in OpenSees with twenty Elastic-Perfectly Plastic non-linear displacement-based beam-column elements with equal length, integrated at 4 points along the element.

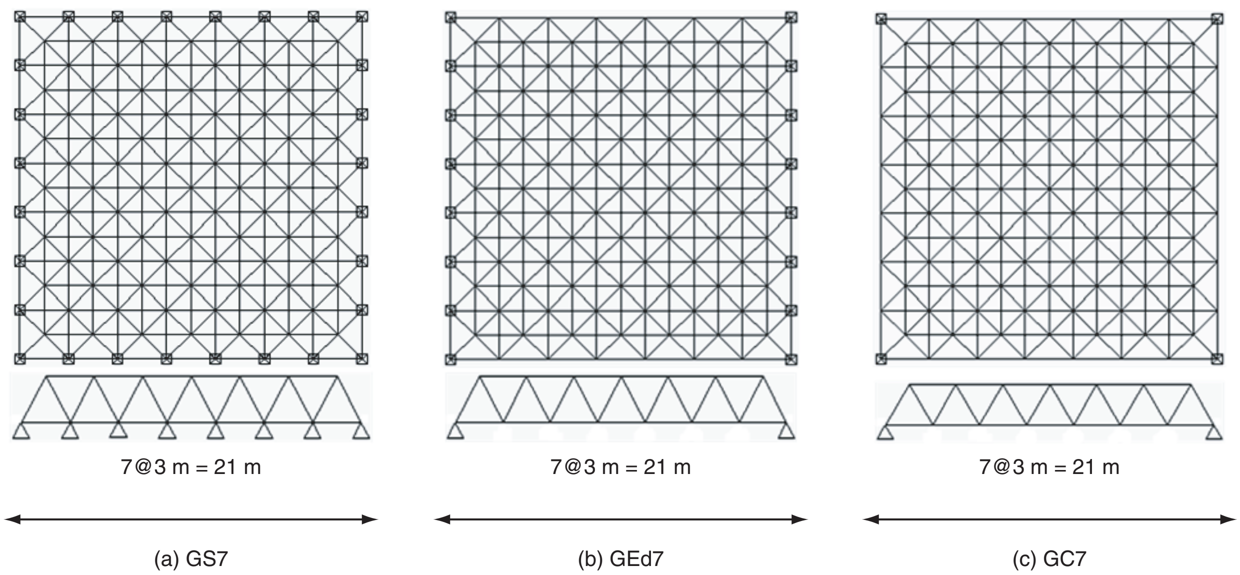

Figure 1. Configuration of the selected double layer grids

Table 1. member size of model grid structures

Members' typology	Type of section (mm)	Area section (mm^2)	Length (m)	Slenderness coefficient
Type 1	CHS 114.3 × 4	1390	3	76
Type 2	CHS 101.6 × 6.3	1890	3	89
Type 3	CHS 88.9 × 6.3	1634	3	102
Type 4	CHS 88.9 × 20	4329	3	118
Type 5	CHS 114.3 × 20	5925	3	88
Type 6	CHS 114.3 × 4	1390	2.5	63
Type 7	CHS 88.9 × 6.3	1634	2.5	85

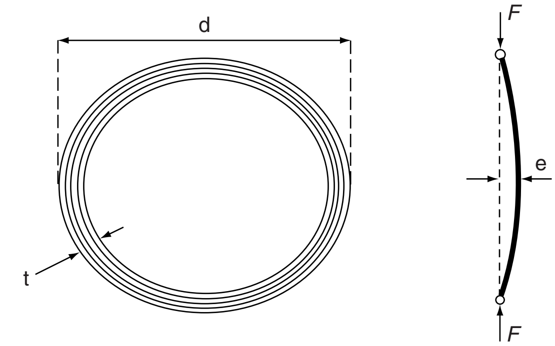

Figure 2. Geometrical and meshing specifications of the compression member model

The integration is based on the Gauss-Legendre quadrature rule which enforces Bernoulli beam assumptions.

All the subjected profiles in this study have been modeled using the "Fiber section" model in the OpenSees software. The cross section of the member along its radius and circumference has been divided to 4 and 16 equal parts respectively (Figure 3). Finally, the axial force-displacement relationship of the model was obtained through displacement control analysis using arc length algorithm with considering both geometric and material non-linearity (Mazzoni *et al*. 2005)

Using this method the model of compression behavior of the different specified members were

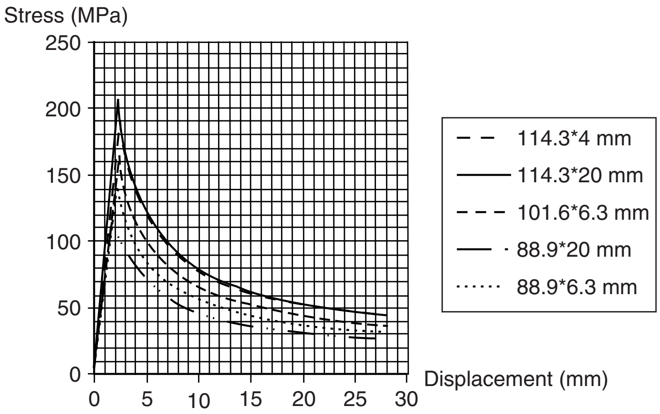

Figure 3. Axial load-axial displacement behavior of the compression members

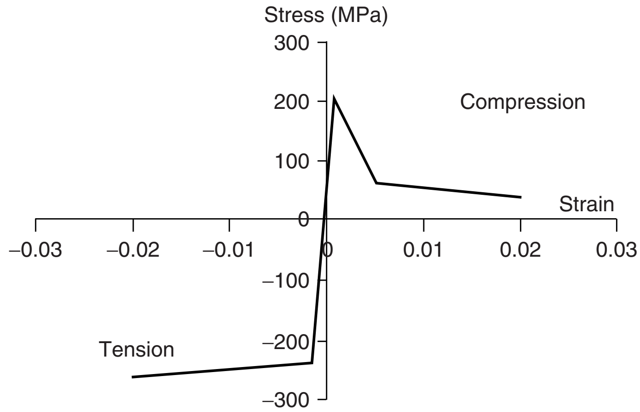

Figure 4. Idealized axial stress-axial strain equation of a sample member in tension and compression

determined, and then using a multi-linear curve as shown in Figure 4, the idealized axial stress- axial strain relationship of each member is defined.

5. RANDOM MODELING OF THE MEMBERS' IMPERFECTIONS

For the subjected double-layer space structure, a random number has been generated as length imperfection for each member of the structure using the normal distribution. This is done to ensure that when the imperfections are randomly distributed amongst the members and they're being analyzed, the normal probability density function governs. The parameters of the normal distribution have been considered in such a way so that they would have an average of zero and a standard deviation of 0.001 times the length of the member. Thus, based on the generated random numbers, member length imperfections for both long and short members have been randomly taken into account. Through the generation of random numbers using the normal distribution and the aforementioned specifications, the probability density function of the imperfections is obtained, as shown in Figure 5.

To model the length imperfection of the members, the method proposed by El-Sheikh (2002) has been employed. As depicted in Figure 6, in order for a member to be in its ideal place, it has to be subjected to

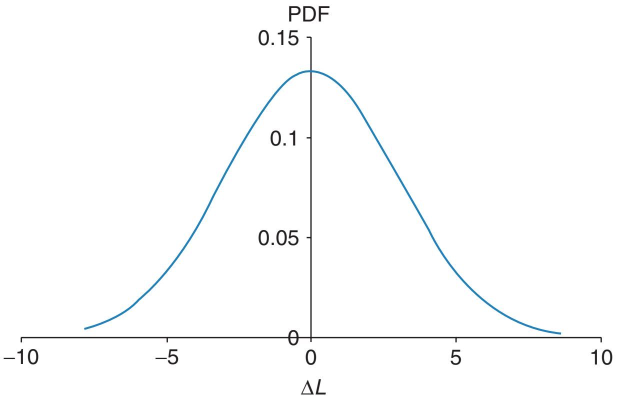

Figure 5. Probability density function of the members' length imperfections in compression members with normal distribution for 3 meter members

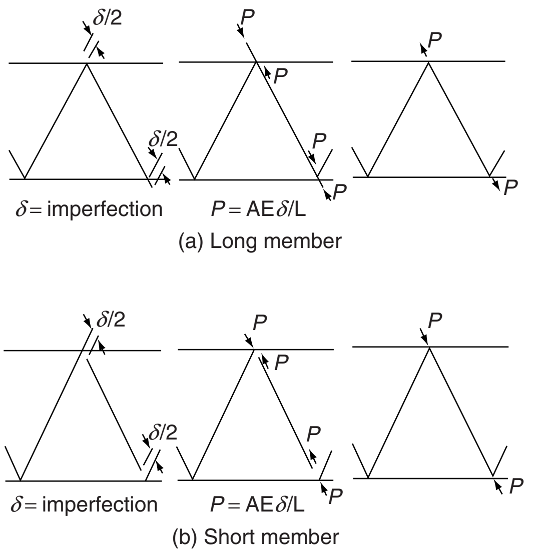

Figure 6. Modeling procedure of the imperfect members

tension or compression. These forces cause a difference in the behavior of the member and will bring about a change in the stress-strain diagram of the imperfect member. Length imperfections, as depicted in Figure 6, have been exerted on the members as force couples on the ends of the members so that they can be placed in their places by becoming longer or shorter. The exerted force on the member in order to cause the length change is given by:

$$P = \frac{EA\Delta L}{L} \tag{1}$$

Where P, E, A and ΔL are the force exerted on the member, the modulus of elasticity, the cross-sectional area and the amount of the imperfection of the member, respectively. These forces will surely affect the stress-strain specifications of the member and to shorten or elongate the member by ± ΔL, a force couple is needed to produce an axial stress-equal to $\sigma = E\varepsilon = E.\Delta L/L$. So, each generated ΔL corresponds to the forces that alter the behavior of the imperfect member. Ultimately, these changes only appear in the idealized stress-strain diagram of the imperfect member. Figure 7 shows the stress-strain diagram of the profile type 2, having taken into account the length imperfection of the member. As it was told, the length imperfection of the members are random and follow the probability density function. In Figure 7, for example, the values –2.4 mm and +2.4 mm have been considered as the imperfections that cause a member to be short or long, respectively.

It is worth mentioning that in this study, for each length imperfection, such calculation has been carried

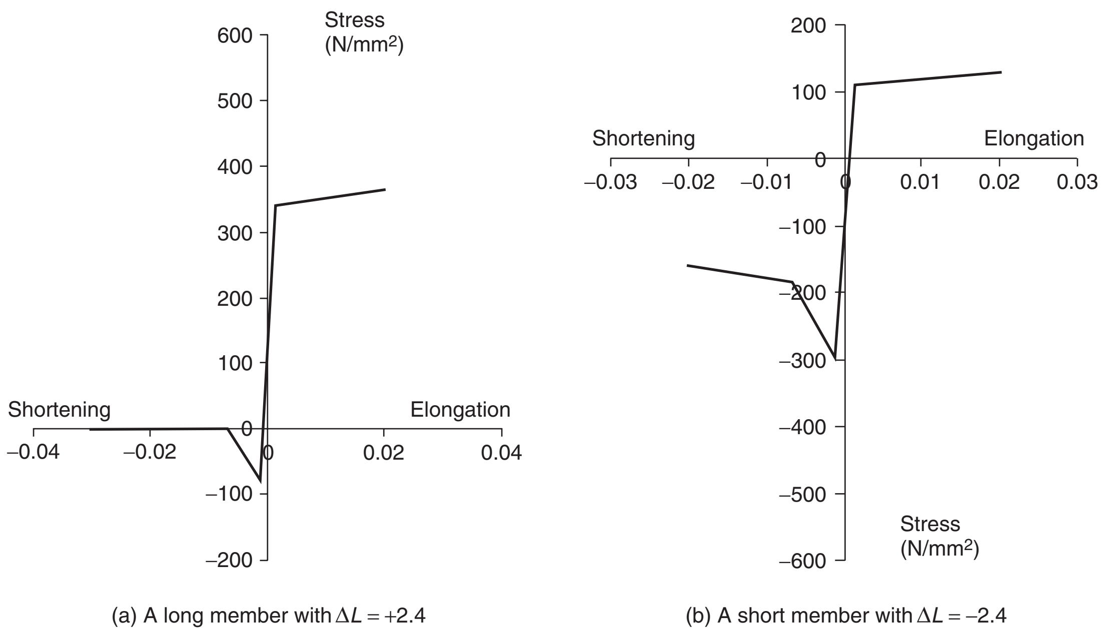

Figure 7. Idealized stress-strain relationship of imperfect members under tension and Compression for the type 2 sample with 3 meter length

out and the stress-strain relationships of the imperfect member, as well as the behavioral model for both long and short members, have been obtained and illustrated, as shown in Figure 7. Afterwards, every one of these stress-strain relationships that correspond to a certain imperfection, is modeled as a random variable in OpenSees software. Thus, for each imperfect member of the grid, a proper stress-strain diagram has been generated using random numbers.

6. RESULTS OF THE RANDOM COLLAPSE ANALYSIS

To calculate the bearing capacity of the imperfect grid, first the amount of the length imperfection of the members has been considered as a random variable with normal distribution. By allocating a random length imperfection to each member, their idealized stress-strain diagrams change and the effect of the length imperfection of the members will have been taken into account by modifying these diagrams. As shown in Table 1, there are 7 types of rods with different lengths and cross-sections in the model structures and it is assumed that each one of them can have an average length imperfection of zero and a standard deviation of 0.001L(L is the member length) with normal distribution. After obtaining the stress-strain diagram of each member based on its imperfection, the push down analysis were performed. In these analyses, both the geometrical and the material nonlinearities have been considered and the diagram of the applied vertical load against the displacement of the middle joint of the lower layer has been obtained. This procedure should be performed multiple times in order to get more reliable results. The obtained results from 1000 simulations have been defined as load-displacement diagrams and are depicted in Figures 8 to 10. For more clarity and recognizing the type of the structure's collapse behavior, some of these diagrams have been presented again in Figure 8 as examples. . The red diagram in Figure 9 presents the force-displacement values for the perfect structure (without any member length imperfections).

According to Figures 8 to 10, if the first peak of each load-displacement curve is considered as the collapse point or the capacity of the system, the statistical distribution of the collapse loads and their respective statistical parameters can be obtained through the required statistical calculations. Based on this, the probability density and the cumulative distribution of the ultimate capacity of these systems have been obtained and are shown in Figures 11, 12 and 13. Statistical specifications of the studied systems such as the average and the standard deviation of the collapse load are presented in Table 2. Also, this table presents the maximum and the minimum capacities that have

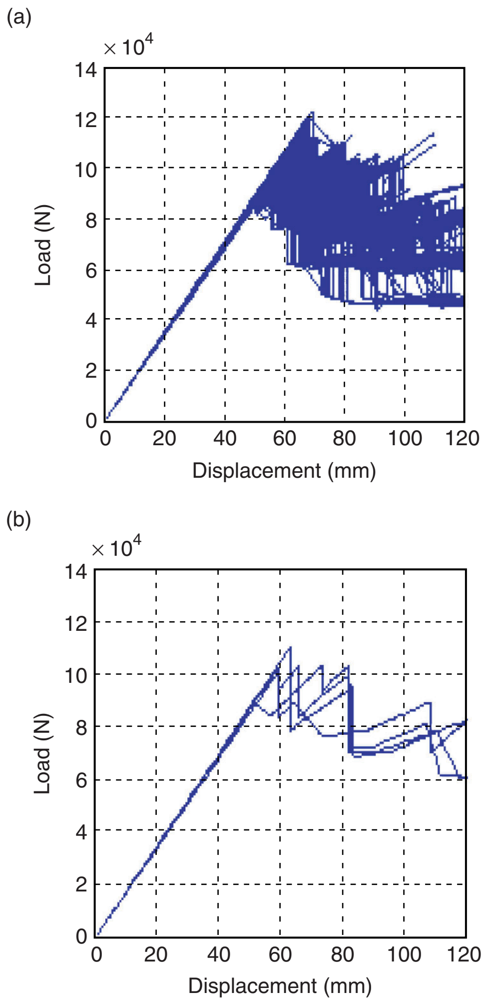

Figure 8. Load -displacement diagrams of the double-layer space structure grids: (a) GEd7 all of the samples; and (b) GEd7 some of the samples

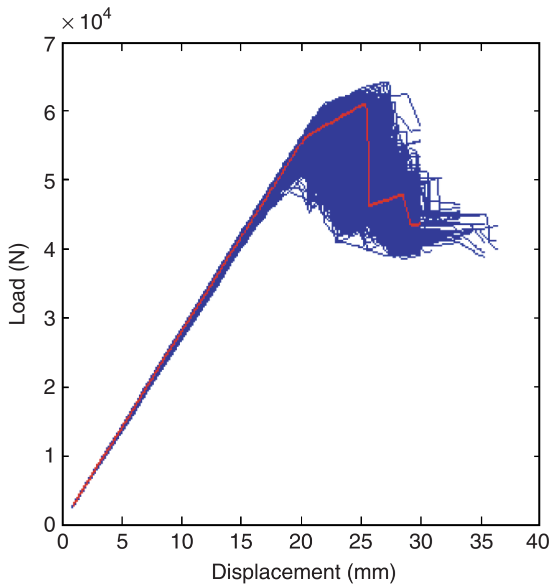

Figure 9. Load -displacement diagrams of the double-layer space structure for GC7 grid

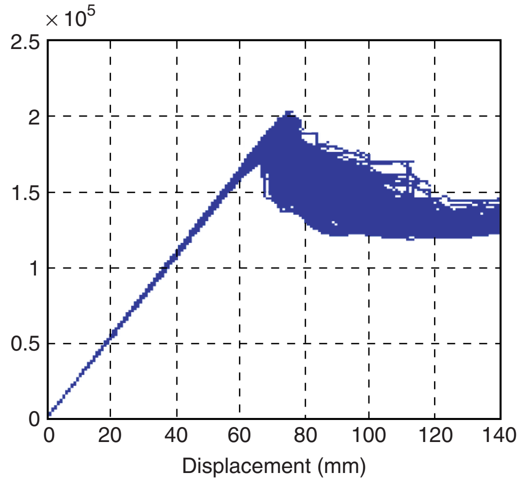

Figure 10. Load -displacement diagrams of the double-layer space structure for GS7 grid

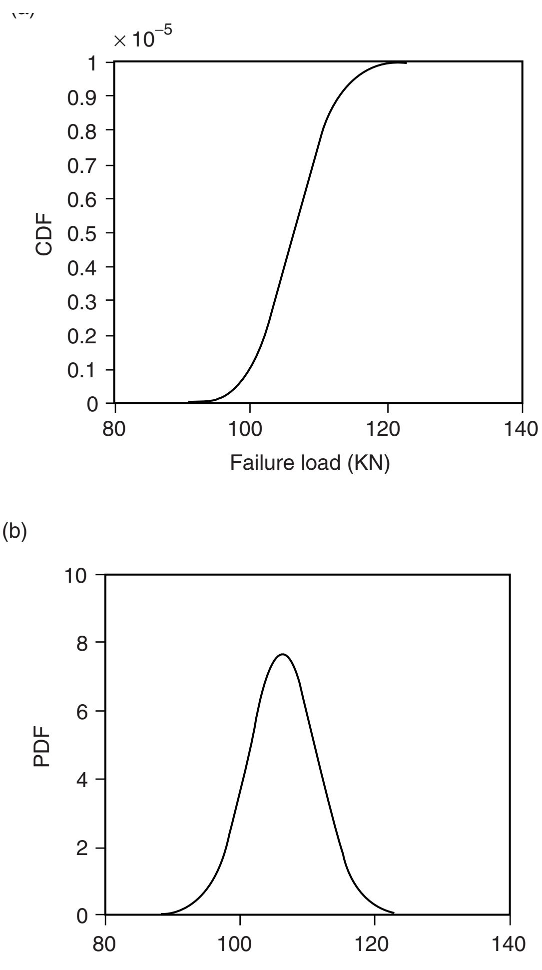

Figure 11. Diagrams of the statistical distribution of the collapse load of GEd7 double-layer grid: (a) Cumulative distribution diagram; and (b) Probability density diagram

been obtained through 1000 random distribution of imperfections among the members.

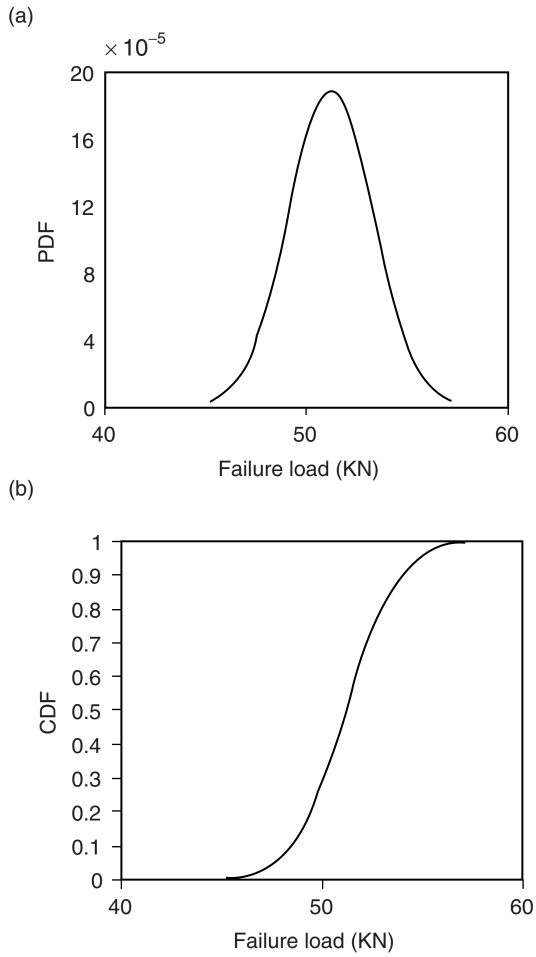

Figure 12. Diagrams of the statistical distribution of the collapse load of GC7 double-layer grid: (a) Probability density diagram; (b) Cumulative distribution diagram

According to the load-displacement diagrams of the GC7, GS7, GEd7 grids and Table 2, it can be seen that the GS7 grid can carry loads three and a half times and one and a half times more than the GC7 and GEd7 grids, respectively. Also, the GEd7 grid's load carrying capacity is two times more than that of the GC7 grid. As it is expected, by increasing the number of supports, the structure's bearing capacity increases and structures with surrounding supports show far greater capacities than those that have corner and edge supports. And when one of the members collapses, there won't be a significant drop in the bearing capacity of the structures because of the multiple alternative paths through which the load can be distributed in the structure.

7. RELIABILITY

Structural reliability is the probability that a system will function desirably under predetermined circumstances for a specified time. Reliability of a structure is commonly shown by R and is defined as $R = 1-P_f$, in which P_f is the probability of the failure of the structure. Since the only

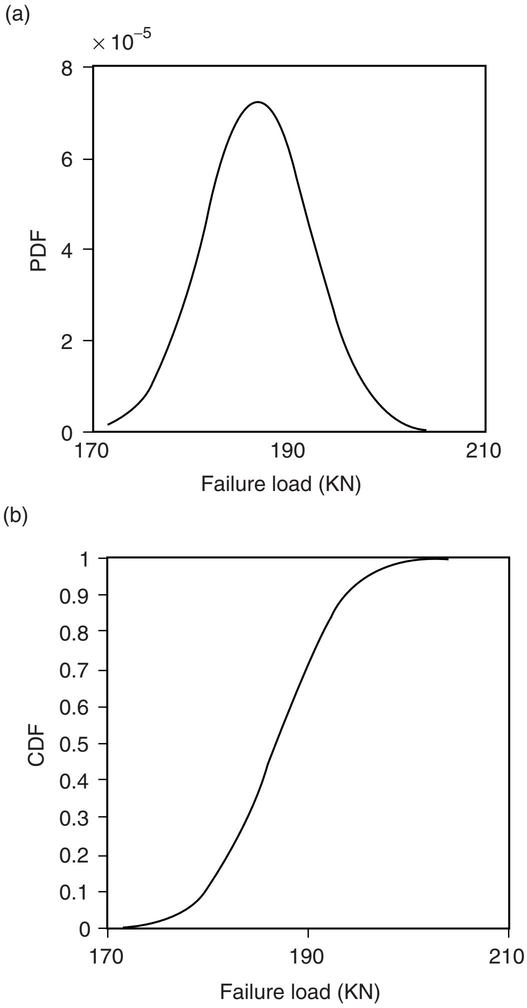

Figure 13. Diagrams of the statistical distribution of the collapse load of GS7 double-layer grid: (a) Probability density diagram; (b) Cumulative distribution diagram

random variable of the problem is the length imperfection of the members in which its effects is applied on the stress-strain diagrams of the imperfect members and that the collapse probability distribution of the structures are obtained as Figures 11(b), 12(b) and 13(b), the reliability of the structures can be defined as follows. The reliability of the structure R at a specific applied load such as F_s is the probability that the system's collapse load F is greater than the mentioned specific load. This can be mathematically expressed as (Mazzoni *et al*. 2005):

$$\mathbf{R}\,(\mathbf{F_s}) = \mathbf{P}\,(\mathbf{F} > \mathbf{F_s}) \tag{2}$$

By using this definition (Eqn 2) and the calculated collapse probability distributions in the previous section, the reliability of the double-layer grids has been determined as shown in Figure 14. It should be noted that in this figure, the phrase "perfect structure" implies a structure without any member length imperfection.

The data that the diagram of Figure 14 present can be provided in a separate table. As an example, the reliability characteristics of the GS7 grid is presented in Table 3. The second column shows the capacity decrease of the structure as a result of random geometrical imperfections. Another column has been added to the table under the name "load factor" and it means that if a designer wants to ensure a specific safety of the structure, he can do so by multiplying the design load by the respective load factor from the table.

According to Figure 14(c) and Table 3, in the design of a system with the reliability of, for example 0.99, it can be seen that the capacity of the imperfect system is equal to 88.9 percent of that of the perfect system, which is 11 percent less than the capacity of the ideal system. In other words, to design a safe system with the reliability of 0.99, the design capacity has to be considered 12 percent greater than the capacity of the perfect structure. Or, for example, to design a system with the reliability of 0.95, the capacity of the imperfect system is equal to 90.4 percent of the capacity of the perfect system which is 9.6 percent less than the capacity of the ideal system and so forth.

In systems, where the ratio of the capacity of the imperfect structure to the capacity of the perfect structure is greater than one, the reliability of the system has been conservatively considered equal to one and essentially the subjected systems are the ones that have experienced a decrease in their capacities due to the random distribution of imperfections. Another point to note is that in some of the random analysis, the mentioned ratio is greater than one and reaches 1.075 at

Table 2. Statistical specifications of the collapse load

Grid's name with member Length imperfections	Failure load (N)					Weight (kg)
	Mean (χ)	Deviation(s)	RSD = σ/χ	P_{MAX}	P_{MIN}	
GC7	52341.8	3143.2	0.006	58142.2	44363.9	8751
GEd7	104372.5	6105.5	0.058	123540	87502.7	7368
GS7	187940.2	5730.5	0.030	206552	170459	6321

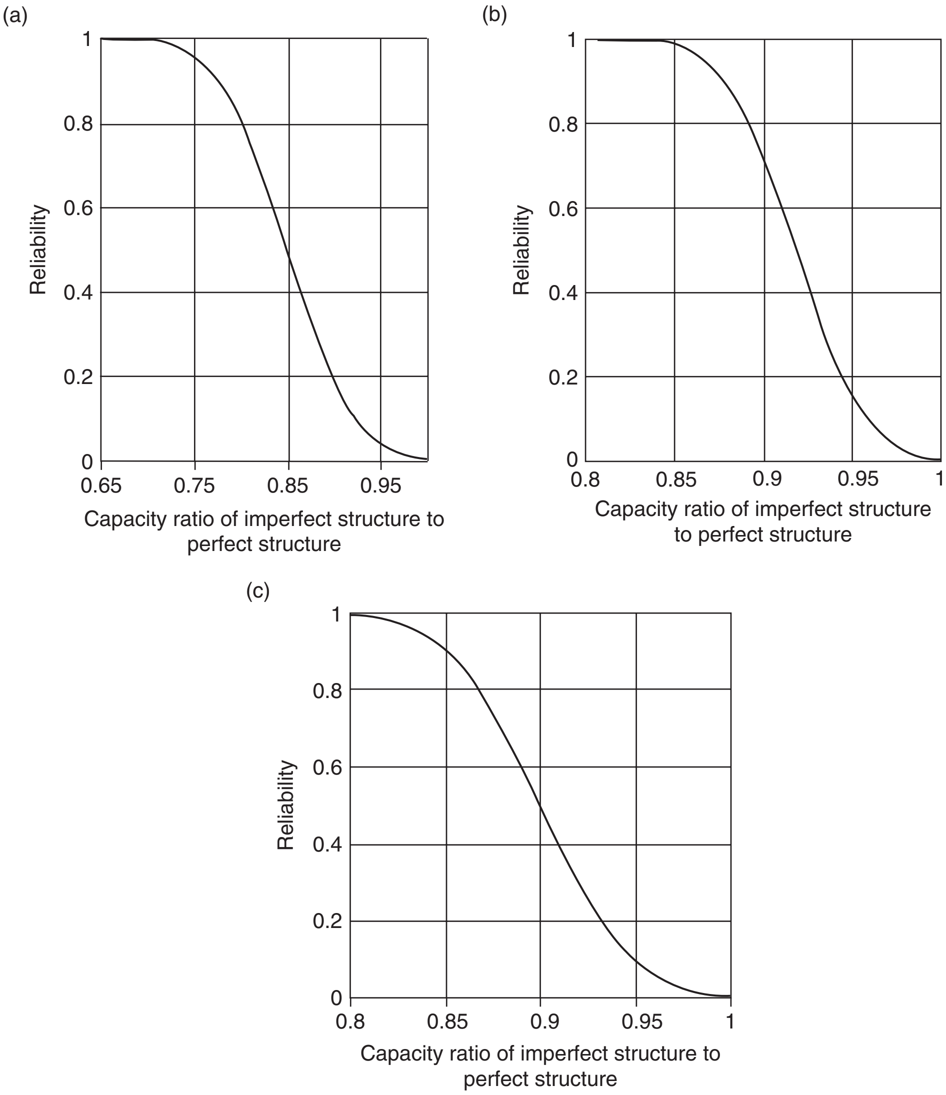

Figure 14. Reliability of the Double layer grids: (a) GEd7; (b) GC7; and (c) GS7

Table 3. Reliability for two layer grid with different supports

	Capacity ratio of imperfect structure to perfect structure			Coefficient load factor		
Reliability	**GS7**	**GC7**	**GEd7**	**GC7**	**GEd7**	**GS7**
1	0.880	0.683	0.721	1.32	1.28	1.14
0.99	0.889	0.723	0.768	1.28	1.23	1.12
0.98	0.895	0.737	0.779	1.26	1.22	1.11
0.97	0.899	0.743	0.786	1.26	1.21	1.11
0.96	0.902	0.756	0.792	1.24	1.21	1.11
0.95	0.904	0.763	0.796	1.24	1.20	1.10
0.94	0.906	0.768	0.800	1.23	1.20	1.10
0.93	0.908	0.773	0.804	1.23	1.20	1.10
0.92	0.910	0.777	0.807	1.22	1.19	1.10
0.91	0.911	0.780	0.810	1.22	1.19	1.10
0.9	0.912	0.784	0.812	1.22	1.19	1.10
0.85	0.918	0.799	0.822	1.20	1.18	1.09
0.8	0.922	0.810	0.831	1.19	1.17	1.08
0.75	0.926	0.820	0.838	1.18	1.16	1.08
0.7	0.929	0.829	0.844	1.17	1.16	1.08

its maximum. In such circumstances, the random distribution of imperfections and the way they're distributed throughout the grid, causes a state of pre-stressing and this results in an increase in the bearing capacity of the structure this is quite evident in Figure 9. The above information can be interpreted as follows. Suppose the aim is designing a double-layer space structure using the diagram of Figure 14(c) or its corresponding data from Table 3. In this case, if for example the desired reliability is 0.98, the required ultimate capacity of the system has to be multiplied by the corresponding load factor from the last column of Table 3 (in this case 1.11).

In addition to this, by knowing the maximum load bearing capacity of the structure, the designer can ensure the safety of the structure under any probable overload and if necessary, adopt the appropriate measures to increase the safety of the structure. Figure 9 shows the force-displacement diagram of the GC7 structure in 1000 analysis of random distribution of imperfections. The red diagram in this figure presents the force-displacement values for the perfect structure (without any member length imperfections). It can be seen that out of 1000 force-displacement diagrams, more than 90 percent of which have experienced a decrease in the bearing capacity compared to that of the perfect structure and less than 10 percent of them show an increase in their capacity compared to the perfect structure. For this reason, in systems where the ratio of the capacity of the imperfect structure to the capacity of the perfect structure is greater than one, the reliability of the system has been considered conservatively equal to one. By rigorously investigating the cause of capacity increase in some space structures, it was observed that in such cases the random distribution of imperfections has been conducted in such a way that the compression and tension members have the imperfections of being short and long, respectively. This yields a maximum capacity increase of 7.5 percent in some cases. Thus, intentionally implementing imperfections in some of the members can significantly heighten the capacity of the space structure. As an example, the horizontal members of the upper layer which are predominantly compression members, can be constructed shorter. In this regard, determining the appropriate values for member length imperfections requires further study and modeling.

8. CONCLUSION

Conducted analyses on a selected offset double-layer grid structures by employing the Monte Carlo simulation method indicate the high sensitivity of these structures to random initial geometrical imperfections. It is demonstrated that by drawing out reliability diagrams (as shown in Figure 14) and applying them in the design of structures, one can easily determine the design load of a structure with desired safety. As a matter of fact, possessing such diagrams help designers in creating designs with consistent levels of reliability without administering detailed reliability analysis for every single design.

The type of the supports plays an undeniable role in the bearing capacity of double-layer grids. Results show that the best types of supports are surrounding supports, edge supports and corner supports, respectively. It should be noted that more supports provide more alternative paths through which the collapse loads can be carried over. It was observed that to achieve a reliability of 99 percent, the capacities of these systems in Figure 14 have been considered equal to 72.4, 76.8 and 88 percent respectively, which are respectively less than the capacities of the perfect systems by the percentages of 28, 23 and 12. Yet again the GS7 grid shows a smaller drop in its capacity and its sensitivity to length imperfections is lesser compared to the other two grids.

Distributing the shortness imperfection amongst the compressional members has caused tensile stresses among these members and that brings about a state of pre-stressing in the grid which in turn increases the load bearing capacity of the member and consequently that of the structure. In this paper, by comparing the reliability of double-layer grids having corner (GC7), edge (GEd7) and surrounding (GS7) supports at the state of pre-stressing, it was observed that the capacities of these systems at their maximum are respectively 5.0, 6.3 and 7.5 percent greater than that of the perfect structure. Therefore, intentionally implementing imperfections in some of the members will cause a significant capacity increase in the space structure.

Finally, it is worth pointing out that generalizing the analytical results of this study to the other types of double-layer grid space structures is not recommended and to determine the reliability of the other types, further studies are required in the same way.

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