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Evaluation of the $B_c^+ \to D^0 K^+$ decay by the factorization approaches and applying the effects of the final state interaction

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Abstract

In this paper the decay of B_c^+ meson which consists of two b and c heavy quarks, into the D^0 and K^+ mesons is studied in two steps. In the first step, the QCD factorization (QCDF) approach is considered in the initial evaluation, the result of calculation is $\mathcal{B}(B_c^+ \to D^0K^+)_{QCDF} = (6.41 \pm 1.77) \times 10^{-7}$. While the available experimental result for this decay is $(f_c/f_u) \times \mathcal{B}(B_c^+ \to D^0K^+) = (9.30 \pm 0.60) \times 10^{-7}$, by applying the theoretical value of the f_c/f_u that span the range of [0.4, 1.2]%, the result for QCDF approach becomes $(f_c/f_u) \times \mathcal{B}(B_c^+ \to D^0K^+)_{QCDF} = (2.56 \pm 0.71) \times 10^{-9} \sim (7.69 \pm 2.12) \times 10^{-9}$and the branching ratio in the experimental observation is obtained in the range of $(3.72 \pm 0.24) \times 10^{-5}$ to $(11.16 \pm 0.72) \times 10^{-5}$. Therefore, it is decided to calculate the theoretical branching ratio by applying the final state interaction (FSI) through the T and cross section channels. In this process, before the B_c^+ meson decays into two final state mesons of D^0K^+, it first decays into two intermediate mesons like $J/\psi D_s^{*+}$, then these two mesons transform into two final mesons by exchanging another meson like D^0. The FSI effects are very sensitive to the changes in the phenomenological parameter which appear in the form factor relation. In most calculations, changing two units in this parameter, makes the final result multiplied in the branching ratio, therefor the decision to use FSI is not unexpected. In this study there are nineteen intermediate states in which the contribution of each one is calculated and summed in the final amplitude. Considering $f_c/f_u = [0.4, 1.2]\%$, the numerical value of the $(f_c/f_u) \times \mathcal{B}(B_c^+ \to D^0K^+)$ is from $(0.47 \pm 0.08) \times 10^{-7}$ to $(13.98 \pm 2.08) \times 10^{-7}$ for which obtained by entering the FSI effects ($\eta = 1 \sim 3$). It should be noted that by choosing the value of the η according to the mass of the exchange meson, as $\eta = 3$ for exchange meson of D^* (or D) and $\eta = 1$ for exchange meson of K^* (or K) the obtained result is $(f_c/f_u) \times \mathcal{B}(B_c^+ \to D^0K^+) = (11.72 \pm 1.36) \times 10^{-7}$, that is in very good agreement with the experimental result.

Keywords: standard model, b meson decays, factorization, final state interaction

1. Introduction

The discovery of B_c^+ meson was first reported by CDF collaboration at the Fermi Lab in 1998, during the process of the $B_c^+ \to J/\psi \ell^+ \nu_\ell$ decay [1]. After that, in 2008 the $B_c^+ \to J/\psi \pi^+$ decay was observed by CDF and D^0 collaborations with 8σ [2] and 5σ [3], respectively. Also the $B_c^+ \to J/\psi \pi^+$ decay in 2013 was observed by LHCb collaboration in proton-proton collision with center of mass energy 7 TeV. The B_c^+ meson is the only meson that has been observed until now that has two heavy quarks with different flavors. Both of these quarks have a strong desire to decay. This meson can not be destroyed to produce a gluon, it can only be decayed by weak interaction. For this reason the meson B_c^+ is a good case to study the mechanism of weak decay of heavy flavors and evaluation of quark-flavor mixing

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in the standard model. The B_c^+ meson has many weak decay channels, all of them can be classified into three different classes:

1. The decay of b-quark into two c or u-quarks, which at this step another c-quark is added as a spectator to the collection.
2. The decay of c-quark into two s or d-quarks which a b-quark used as a spectator.
3. Weak annihilation decay channel.

For the class 1, the $|V_{ub}|$ and $|V_{cb}|$ matrix elements of CKM matrix elements are of interest.

For the class 2, one of the heavy mesons B_s or B_d is in the final state . It has significant effects for the c-quark decay in the phase space. In this class, $|V_{cs}|$ and $|V_{cd}|$ of CKM matrix elements are of interest, in comparison with the elements $|V_{ub}|$ and $|V_{cb}|$ used in class 1, have much larger values.

In the class 3, the weak annihilation decay of B_c^+ meson has significant amount compared with the B_u^+ decay so that the ratio of $|V_{cb}|^2$ to $|V_{ub}|^2$ in weak annihilation decay of B_c^+ and B_u^+, is approximately 100. In fact, we have $|V_{cb}|^2/|V_{ub}|^2 \sim 100$. This means that unlike B_u^+ decay, which can be ignored, the process of annihilation decay has a significant value in B_c^+ decay. In the study of B_c^+ meson decay, all three classes are very important. Unlike B_u^+ and B_d^0 mesons decay, more than 70% of the B_c^+ meson decays occur by c-quark decay. The transition $c \to s$ has been observed through the $B_c^+ \to B_s^0\pi^+$. The b-quark decay hold only near to 20% of B_c^+ meson decays. In the case that there is no c-quark in the final state, the $\bar{b}c \to W^+ \to \bar{q}q$ annihilation amplitudes are only 10% of the total B_c^+ meson decays. Before the decay of $B_c^+ \to D^0K^+$ was observed in experiment, it's branching ratio was calculated using perturbative quantum chromodynamics (pQCD) method, which has been obtained by the value of 6.60×10^{-5} [4]. Also, it has been calculated using the factorization approach. The result of this calculation is 1.34×10^{-7} [5] which is in good agreement with what we have achieved in this method, the decay $B_c^+ \to D^0K^+$ has been observed by LHCb collaboration, they have obtained B_c^+ production compared to B_u^+ as [6]:

$$\frac{f_c}{f_u}\mathcal{B}(B_c^+ \to D^0K^+) = (9.3^{+2.8}_{-2.5} \pm 0.6) \times 10^{-7}, \qquad (1)$$

in which, the ratio f_c/f_u is an unknown value obtained by evaluating two decays $B_c^+ \to J/\psi\pi^+$ [7, 8] and $B_c^+ \to J/\psi K^+$ [9] in the range of 0.004 to 0.012 [10]. In this case, the branching fraction in the experimental observation becomes:

$$\mathcal{B}(B_c^+ \to D^0K^+) = (3.72^{+1.12}_{-1.00} \pm 0.24) \times 10^{-5} \sim (11.16^{+3.36}_{-3.00} \pm 0.72) \times 10^{-5}. \qquad (2)$$

It is clear that the factorization approach is about 100 times smaller than experience, while the pQCD result is within the range obtained from equation (2). It should be noted that the branching fraction in the experimental observation comes from experience and theory, so comparing the result of pQCD with the values of equation (2) is not accurate by itself, it must also be compared with equation (1). This comparison has not been made to the experimental value in [4]. Also the result of pQCD dose not cover the range of the branching ratio in equation (2). In [5], unfactorizable contributions are not considered, which reflects the fact that the heavy observed gluon contributions in strong interactions are neglected. By doing this, it will not matter how other parameters like strong phase and phenomenological parameter can be adjusted. The phenomenological parameter is a parameter that appears in the FSI form factors which increases the strong interaction contribution. The final results are very sensitive to this parameter so that, the range of final results with a little change in phenomenological parameter, changes dramatically.

In [6], it is explicitly stated that this decay is expected to continue with penguin and weak annihilation amplitudes but, as we know, the main contribution of the final amplitude lies in the tree amplitudes. In fact, if we remove the tree amplitude contribution from what was done in [5], the result will be 10^3 times smaller than the experience. By entering unfactorizable contribution, one can not compensate 10^3 times smaller than the result. It seems, is needed a model, a method or applying natural effects to compensate this major difference, we enter FSI effects. As it was said, the experimental range of $B_c^+ \to D^0K^+$ decay is within the range of 3.72×10^{-5} to 11.16×10^{-5}, that is, a relatively large range, this is why we decided to re-calculate the branching ratio of this decay by entering FSI effects. In the previous works [11–13], we have seen that the calculation of intermediate state effects is very sensitive to phenomenological parameter. In some cases, by changing the two units in the value of this parameter, the final result changes several times. In the $B_c^+ \to D^0K^+$ decay, since the range of upper limit in experimental result is about three times more than it's lower limit, therefore we expect that the phenomenological parameter change can cover the range of experimental result. In this paper, we are talking about the fact that during the $B_c^+ \to D^0K^+$ decay some middle particles are produced, in the way that before D^0 and K^+ particles occur in the final state, some middle particles are formed which have been converted into final mesons by exchanging another particle. The process of producing these middle and exchanged particles is determined through Feynman diagrams.

For FSI quark model, the Feynman graphs are presented in two types: the first one is the T-channel and the second one is the cross section-channel. In the T-channel, two final mesons of D^0 and K^+ share one quark and one anti-quark with the same flavor (u). The intermediate mesons are produced by sharing c, d and s quarks. In this case, the intermediate state mesons $J/\psi D_s^{+(*)}$ (both $J/\psi D_s^+$ and $J/\psi D_s^{*+}$), $D^{+(*)}K^{0(*)}$ and $D_s^{+(*)}\phi$ can be produced with D^0, π^- and K^- exchange mesons, respectively. In the cross section channel, two final mesons exchange one non-flavored quark with intermediate mesons crosswisely. For the $B_c^+ \to D^0K^+$ decay, the two final state mesons D^0 and K^+ exchange c and u quarks with intermediate mesons, crosswisely. In this process, $D^{+(*)}P$ and D^0 mesons can be present as intermediate and exchange mesons, respectively. Here, P can be π^0, ρ^0, η and ω. Also,

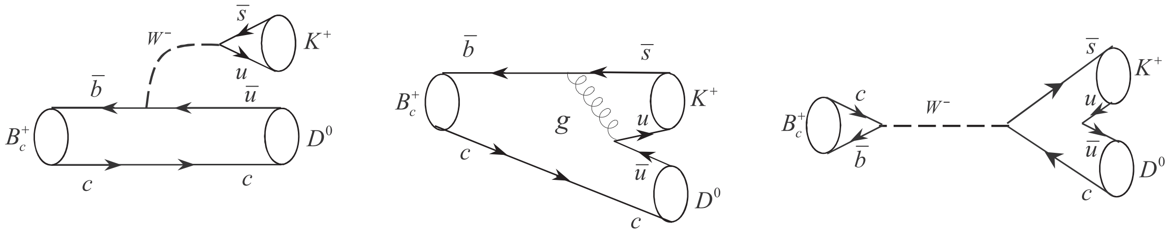

Figure 1. Diagrams for $B_c^+ \to D^0K^+$ decay.

each of two final mesons D^0 and K^+ can exchanges one anti-quark $\bar{u}$ or $\bar{s}$ with intermediate state mesons, crosswisely. Then, intermediate mesons will be the same as the previous mesons $D^{+(*)}P$ (which p-types have already been identified). The exchanged meson in this process is K^-. In general, the $B_c^+ \to D^0K^+$ decay is transformed into following decays using FSI effects on the T-channel:

$$B_c^+ \to J/\psi D_s^{+(*)} \to D^0K^+ \text{ exchange meson is } D^0,$$
$$B_c^+ \to D^{+(*)}K^{0(*)} \to D^0K^+ \text{ exchange meson is } \pi^-,$$
$$B_c^+ \to D_s^{+(*)}\phi \to D^0K^+ \text{ exchange meson is } K^-, \quad (3)$$

and transformed into the following decays in the cross section-channel:

$$B_c^+ \to D_s^{+(*)}P \to D^0K^+ \text{ exchange meson is } D^0,$$
$$B_c^+ \to D_s^{+(*)}P \to D^0K^+ \text{ exchange meson is } K^-. \quad (4)$$

As an example, consider the intermediate state $B_c^+ \to J/\psi D_s^{+(*)}$. In this process, first, two mesons J/ψ and $D_s^{+(*)}$ are produced in the intermediate state. They are then converted to the final states D^0 and K^+ by exchanging a D^0 meson. To calculate the total amplitude of the $B_c^+ \to D^0K^+$ decay, the five channels listed above (three T-channels and two cross section-channels) which are calculated using FSI method, should be added. In the calculations, we also need the individual amplitudes of the intermediate states. So, in the next section, we calculate the intermediate state amplitudes.

2. Short distance processes

2.1. Amplitude of $B_c^+ \to D^0K^+$ decay by using the QCD factorization approaches

A detailed discussion of the QCD factorization approaches can be found in [14–17]. Factorization is a property of the heavy-quark limit, in which we assume that the b quark mass is parametrically large. The QCD factorization formalism allows us to compute systematically the matrix elements of the effective weak Hamiltonian in the heavy-quark limit for certain two-body final states D^0K^+. In this section, we obtain the amplitude of $B_c^+ \to D^0K^+$ decay using QCD factorization method. Under the factorization approach, there are color-allowed tree and suppressed penguin diagrams to $B_c^+ \to D^0K^+$ decay. We adopt leading order of Wilson coefficients at the scale m_b for QCD factorization approach. The diagrams describing this decay are shown in figure 1. According to the QCD factorization, the amplitude of $B_c^+ \to D^0K^+$ decay is given by

$$M(B_c^+ \to D^0K^+)_{QCDF} = \frac{G_F}{\sqrt{2}} f_{K^+}[(m_{B_c^+}^2 - m_{D^0}^2)F_0^{B_c^+\to D^0}(m_{K^+}^2) \times (a_1 V_{ub}^* Vus + a_4 V_{tb}^* V_{ts}) + f_{B_c^+} f_{D^0} b_2 V_{cb}^* V_{cs}], \quad (5)$$

where a_1 and a_4 correspond to the current-current tree and penguin, and b_2 corresponds to the current-current annihilation coefficients that are given by

$$a_1 = c_1 + \frac{c_2}{N_c}, \quad a_4 = c_4 + \frac{c_3}{N_c},$$
$$b_2 = \frac{C_F}{N_c^2} c_2 A_1^i, \quad (6)$$

c_i are the Wilson coefficients, $N_c = 3$ is the color number and

$$A_1^i = 2\pi\alpha_s\left[9\left(X_A - 4 + \frac{\pi^2}{3}\right) + r_\chi^{D^0} r_\chi^{K^+} X_A^2\right],$$
$$C_F = \frac{N_c^2 - 1}{2N_c}, \quad (7)$$

where $r_\chi^{D^0} = 1.85$ and $r_\chi^{K^+} 1.14$. For the running coupling constant, at two loop order (NLO) the solution of the renormalizaton group equation can always be written in the form

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2}{\Lambda_{QCD}^2}}\left[1 - \frac{\beta_1}{\beta_0^2} \frac{\ln\left(\ln \frac{\mu^2}{\Lambda_{QCD}^2}\right)}{\ln \frac{\mu^2}{\Lambda_{QCD}^2}}\right], \quad (8)$$

here

$$\beta_0 = \frac{11N_c - 2n_f}{3}, \quad \beta_1 = \frac{34N_c^2}{3} - \frac{10N_c n_f}{3} - 2C_F n_f, \quad (9)$$

and running $\alpha_s(\mu)$ evaluated with $n_f = 5$. There are large theoretical uncertainties related to the modeling of power corrections corresponding to weak annihilation effects, we parameterize these effects in terms of the divergent integrals X_A (weak annihilation)

$$X_A = (1 + \rho e^{i\phi}) \ln \frac{m_{B_c^+}}{\Lambda_h}, \quad (10)$$

where $\rho = 0.5$, $\phi = -55^\circ$ and $\Lambda_h = 0.5$ GeV.

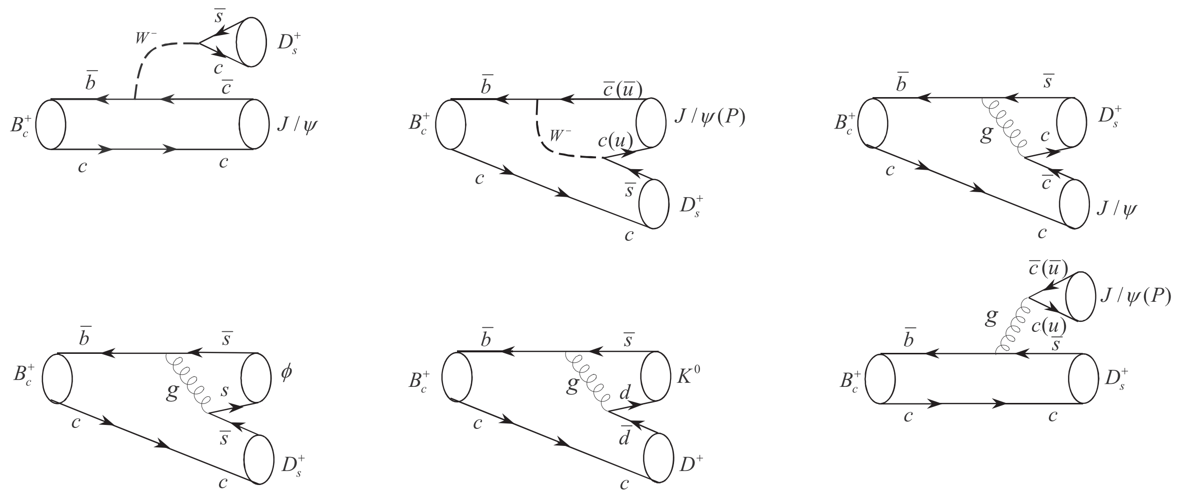

Figure 2. Feynman diagrams for intermediate states decays.

2.2. Decay amplitudes of intermediate states

Each decay in the intermediate states is a two body decay of B_c^+ meson that until now the amplitude of such decays has been achieved in many ways: naive factorization, QCD factorization, improved factorization and using QCD perturbation. In present section, we use QCD factorization to obtain intermediate state amplitudes.

The first intermediate state decay is: $B_c^+ \to J/\psi D_s^{+(*)}$. Usually, in FSI, intermediate amplitudes have dominant contribution. However, here we consider all of them. The decay $B_c^+ \to J/\psi D_s^+$ happens both through $\bar{b} \to \bar{c}$ and $\bar{b} \to \bar{s}$ transitions. The tree transition of $\bar{b} \to \bar{c}$ concludes two contributions of Wilson's coefficients: a_1 and a_2. The matrix elements of both contributions are $V_{cb}^* V_{cs}$. For a_1 contribution, the mesons of J/ψ and D_s^+ place in form factor and decay constant, respectively. But, in a_2 contribution, conversely.

The penguin transition $\bar{b} \to \bar{s}$ has a_3 and a_4 Wilson contributions. In a_3 contribution, mesons of D_s^+ and J/ψ place in the form factor and decay constant, respectively. But, in the a_4 coefficient the rule is the opposite of this. Corresponding matrix elements are $V_{tb}^* V_{ts}$.

The second and third intermediate state decays are $B_c^+ \to D_s^{+(*)}\phi$ and $B_c^+ \to D^{+(*)}K^{0(*)}$. These decays only have penguin transition. In fact, they have just a_4 contribution in which mesons of D_s^+ and D^+ place in form factor and mesons of ϕ and K^0 place in decay constant. Their CKM matrix elements are $V_{tb}^* V_{ts}$.

The fourth and fifth intermediate state decays are the decays of $B_c^+ \to D_s^{+(*)}P$ with $P = \pi^0, \rho^0, \omega, \eta$ which include the a_2 and a_3 contributions of tree transition $\bar{b} \to \bar{u}$ and penguin transition $\bar{b} \to \bar{s}$, respectively. The corresponding matrix elements are $V_{ub}^* V_{us}$ and $V_{tb}^* V_{ts}$, respectively. The appearance of the five decays mentioned above can be seen in the figure 2.

In all these decays, there were contributions of Wilson. These contributions are derived from the combination of Wilson coefficients. If these coefficients are used in the normal form, the factorization is called naive factorization, while, using Wilson effective coefficients, it is called QCD factorization. Wilson tree and penguin contributions get form $a_{1,2} = c_{1,2} + c_{2,1}/3$ and $a_{3,4} = c_{3,4} + c_{4,3}/3$, respectively. The c_i are normal Wilson coefficients, these become Wilson effective coefficients if the vertex correction and hard gluon scattering are taken into account ($c_i \to c_i^{eff}$). In all of these decays, we used terms like: tree transition, penguin transition, decay constant and form factor. The form factors and decay constants for pseudo scalars, D_s^+, D^+, K^0, π^0, η and vector, D_s^{*+}, ϕ, ρ^0, ω mesons can be written, respectively as [14]:

$$
\begin{aligned}
\langle P(p')|V_\mu|B_c(p)\rangle &= \left[(p+p')_\mu - \frac{m_{B_c}^2 - m_P^2}{q^2}q_\mu\right]F_1(q^2) \\
&\quad + \frac{m_{B_c}^2 - m_P^2}{q^2}q_\mu F_0(q^2) \\
\langle 0|A_\mu|P(q)\rangle &= if_p q_\mu \\
\langle V(\epsilon, p')|V_\mu - A_\mu|B_c(p)\rangle &= (\epsilon^* \cdot q)\frac{2m_V}{q^2}q_\mu A_0(q^2) \\
&\quad + (m_{B_c} + m_V)\left[\epsilon_\mu^* - \frac{\epsilon^* \cdot q}{q^2}q_\mu\right]A_1(q^2) \\
&\quad - \frac{\epsilon^* \cdot q}{m_{B_c} + m_V}\left[(p+p')_\mu - \frac{m_{B_c}^2 - m_V^2}{q^2}q_\mu\right]A_2(q^2) \\
\langle 0|V_\mu|V(\epsilon, q)\rangle &= if_V m_V \epsilon_\mu,
\end{aligned}
\tag{11}
$$

where P and V are pseudo scalar and vector mesons, respectively. The q parameter is the four-momentum of propagator the square of which is $q^2 = m_{B_c}^2 + m_p^2 - 2m_{B_c}p_p^0$. For the $B_c^+ \to J/\psi$ transition the form factor is as follows [18]:

$$
\begin{aligned}
\langle J/\psi(\epsilon, p')|V_\mu - A_\mu|B_c(p)\rangle &= f_0(q^2)(m_{B_c} + m_{J/\psi})\epsilon_\mu^* \\
&\quad - \frac{f_+(q^2)}{(m_{B_c} + m_{J/\psi})}(\epsilon^* \cdot p)(p+p')_\mu \\
&\quad - \frac{f_-(q^2)}{(m_{B_c} + m_{J/\psi})}(\epsilon^* \cdot p)(p-p')_\mu
\end{aligned}
\tag{12}
$$

In the equations (11) and (12), $F_{0,1}(q^2)$, $A_{0,1,2}(q^2)$ and $f_{0,+,-}(q^2)$ are the transition form factors. So, amplitudes of

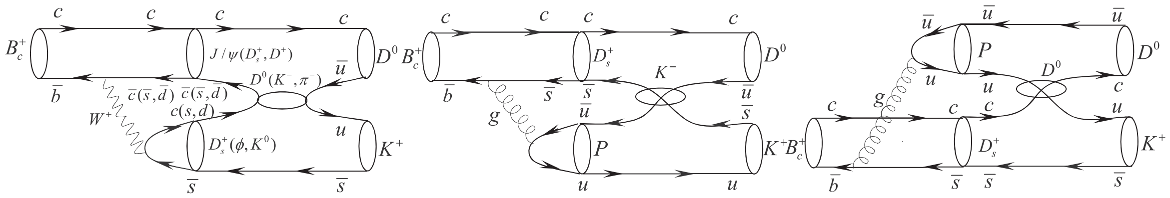

Figure 3. Diagrams of meson vertexes.

intermediate decays take the following form:

$$M(B_c^+ \to J/\psi D_s^+) = i\sqrt{2}\, G_F(\epsilon_{J/\psi}\cdot p_B)\Bigg\{ V_{cb}^{*}V_{cs} \times [a_1 f_{D_s^+}\Bigg((m_{B_c^+} + m_{J/\psi}) f_0^{B_c^+\to J/\psi}(m^2_{D_s^+}) - (m_{B_c^+} - m_{J/\psi}) f_+^{B_c^+\to J/\psi}(m^2_{D_s^+}) - \frac{m^2_{D_s^+}}{(m_{B_c^+} + m_{J/\psi})} f_-^{B_c^+\to J/\psi}(m^2_{D_s^+})\Bigg) + a_2 m_{J/\psi} f_{J/\psi} F_1^{B_c^+\to D_s^+}(m^2_{J/\psi})] + V_{tb}^{*}V_{ts}\Bigg[a_3 m_{J/\psi} f_{J/\psi} F_1^{B_c^+\to D_s^+}(m^2_{J/\psi}) + a_4 f_{D_s^+}\Bigg((m_{B_c^+} + m_{J/\psi}) f_0^{B_c^+\to J/\psi}(m^2_{D_s^+}) - (m_{B_c^+} - m_{J/\psi}) f_+^{B_c^+\to J/\psi}(m^2_{D_s^+}) - \frac{m^2_{D_s^+}}{(m_{B_c^+} + m_{J/\psi})} f_-^{B_c^+\to J/\psi}(m^2_{D_s^+})\Bigg)\Bigg]\Bigg\}$$

$$M(B_c^+ \to J/\psi D_s^{*+}) = i\frac{G_F}{\sqrt{2}}\Bigg\{ f_{D_s^{*+}} m_{D_s^{*+}} \times \Bigg[(\epsilon_{J/\psi}\cdot\epsilon_{D_s^{*+}})(m_{B_c^+} + m_{J/\psi}) f_0^{B_c^+\to J/\psi}(m^2_{D_s^{*+}}) - (\epsilon_{J/\psi}\cdot p_{B_c^+})(\epsilon_{D_s^{*+}}\cdot p_{B_c^+}) \frac{2 f_+^{B_c^+\to J/\psi}(m^2_{D_s^{*+}})}{m_{B_c^+} + m_{J/\psi}}\Bigg] \times (a_1 V_{cb}^{*}V_{cs} + a_4 V_{tb}^{*}V_{ts}) + f_{J/\psi} m_{J/\psi}\Bigg[(\epsilon_{J/\psi}\cdot\epsilon_{D_s^{*+}}) \times (m_{B_c^+} + m_{D_s^{*+}}) A_1^{B_c^+\to D_s^{*+}}(m^2_{J/\psi}) - (\epsilon_{J/\psi}\cdot p_{B_c^+})(\epsilon_{D_s^{*+}}\cdot p_{B_c^+}) \frac{2A_2^{B_c^+\to D_s^{*+}}(m^2_{J/\psi})}{m_{B_c^+} + m_{D_s^{*+}}}\Bigg] a_2 V_{cb}^{*}V_{cs}\Bigg\}$$

$$M(B_c^+ \to D_s^{*+}\phi) = i\frac{G_F}{\sqrt{2}}\Bigg[(\epsilon_{D_s^{*+}}\cdot\epsilon_\phi)(m_{B_c^+} + m_{D_s^{*+}}) A_1^{B_c^+\to D_s^{*+}}(m_\phi^2) - (\epsilon_\phi\cdot p_{B_c^+})(\epsilon_{D_s^{*+}}\cdot p_{B_c^+}) \frac{2A_2^{B_c^+\to D_s^{*+}}(m_\phi^2)}{m_{B_c^+} + m_{D_s^{*+}}}\Bigg] a_4 V_{tb}^{*}V_{ts}$$

$$M(B_c^+ \to D_s^+\phi) = i\sqrt{2}\, G_F f_\phi m_\phi(\epsilon_\phi\cdot p_B) \times F_1^{B_c^+\to D_s^+}(m_\phi^2) a_4 V_{tb}^{*}V_{ts}$$

$$M(B_c^+ \to D^+K^0) = i\frac{G_F}{\sqrt{2}} f_{K^0}(m^2_{B_c^+} - m^2_{D^+}) \times F_0^{B_c^+\to D^+}(m^2_{K^0}) a_4 V_{tb}^{*}V_{ts}$$

$$M(B_c^+ \to D_s^+P) = i\frac{G_F}{\sqrt{2}} f_P(m^2_{B_c^+} - m^2_{D_s^+}) \times F_0^{B_c^+\to D_s^+}(m_P^2)(a_2 V_{ub}^{*}V_{us} + a_3 V_{tb}^{*}V_{ts}), \qquad (13)$$

with $P = \pi^0,\ \rho^0,\ \omega,\ \eta$; $a_2 = c_2 + c_1/3$ and $a_3 = c_3 + c_4/3$.

3. Amplitudes of the long distance processes

We know the above-mentioned decays as intermediate state decays. In this process, first the decaying meson decays into two intermediate mesons, then, these two intermediate mesons are changed into final mesons by exchanging a meson. FSI is done using two channels named T-channel and cross section-channel. In T-channel, the two final mesons share co-flavour quark and anti-quark with their same flavour. But, in cross section channel, two final mesons exchange two non-flavoured quarks or anti-quarks, crosswisely. As we know, the D^0 and K^+ mesons constructed from $c\bar{u}$ and $u\bar{s}$ quark anti-quark, respectively. Then, these two mesons, which are in the final state, can share quark anti-quark with their same flavour i.e. u quark in channel T. On the other hand, intermediate state mesons can be produced by sharing co-flavoured quark anti-quark like c, s and d which can be $J/\psi D_s^{+(*)}$, $D_s^{+(*)}\phi$ and $D^{+(*)}K^{0(*)}$ mesons. Interchanging mesons of these processes are called D^0, K^- and π^-, respectively. In cross section channel, two final state mesons D^0 and K^+ exchange $\bar{u}$ and $\bar{s}$ anti-quarks, respectively. In this state, D_s^+P with $P = \omega$, ρ^0, η and π^0 are intermediate mesons and K^- is interchange meson. In this channel, there is another state in which two final mesons D^0 and K^+ mesons exchange c and u quarks, respectively. Thus, intermediate mesons are

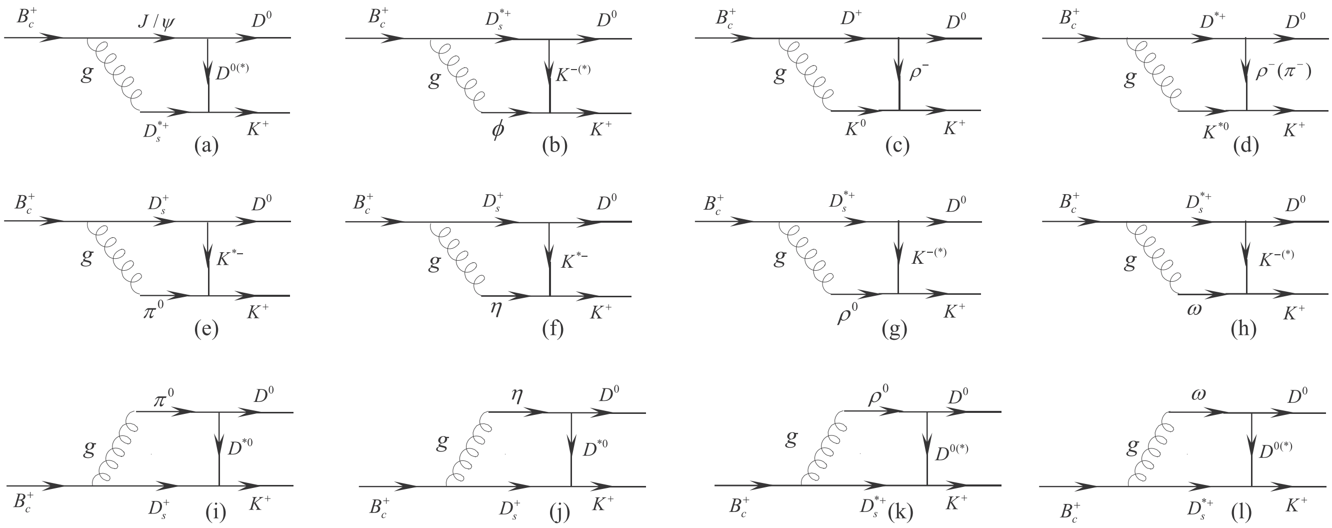

Figure 4. Diagrams of meson vertexes.

the same as the previous i.e. D_s^+p and D^0 is the intermediate meson. Figure 3 illustrates FSI diagrams for $B_c^+ \to D^0K^+$ decay through T and cross section channels. The form factors corresponding to FSI are different from those defined in the previous section, the exchanged particle in interaction i.e. propagator, boson or gluon, but in FSI, the exchange particle is a meson. Since, the form factor depends on mass and momentum of particle, here too, we introduce the FSI form factor as a function of mass and momentum $F(q^2, m_i^2) = (\Lambda^2 - m_i^2)/(\Lambda^2 - q^2)$ in which m_i and q are the mass and momentum parameter that show the effectiveness of strong interaction in a weak interaction through $\Lambda = m_i + \eta\Lambda_{QCD}$. The parameter Λ_{QCD} is strong interaction energy scale, which has the range of from 225 MeV to 750 MeV. We usually consider the value of Λ_{QCD} constant and equal to 225 MeV. Then, we change the phenomenological parameter η. The range of this parameter is defined according to the exchanged mesons. In [19], D and D^* are exchanged mesons which the authors have considered the range of η parameter from 0.5 to 3. However, in [20], for similar exchanged mesons, authors have considered 5 for parameter η. In [21], calculations are made for $\eta = 4$. In present paper, we have considered the range of η from 1 to 3. The more the value of η, the more the effect of strong interaction. The mesons vertex factor is another important factor in FSI. This factor is proportional to the coupling constant of meson in vertices. There are three mesons in the top vertex and three mesons in the down vertex which should follow meson vertex rules. The first of them says that there must be at least one vector meson in each vertex. Also, vector mesons should be symmetric in the final and intermediate state in the manner that if two final mesons are pseudo scalar, the intermediate mesons both should either be pseudo scalar or vector mesons. In the decay considered in this article, $B_c^+ \to D^0K^+$, because two final mesons are pseudo scalar, the intermediate mesons both should be either pseudo scalar or vector mesons. In the case in which both intermediate mesons are pseudo scalar, the exchanged meson should be vector meson. In the case in which both intermediate mesons are vector , the exchanged mesons can be both scalar and vector. Applying these rules, the following decays in the T-channel can be calculated:

$$
\begin{aligned}
&B_c^+ \to J/\psi D_s^{*+} \to D^0K^+ \text{ exchange mesons are } D^0, D^{*0}\\
&B_c^+ \to D^+K^0 \to D^0K^+ \text{ exchange meson is } \rho^-\\
&B_c^+ \to D^{*+}K^{*0} \to D^0K^+ \text{ exchange mesons are } \pi^-, \rho^-\\
&B_c^+ \to D_s^{*+}\phi \to D^0K^+ \text{ exchange mesons are } K^-, K^{*-}.
\end{aligned}
\tag{14}
$$

Also in the cross section channel the following decays are calculated:

$$
\begin{aligned}
&B_c^+ \to D_s^+\pi^0 \to D^0K^+ \text{ exchange meson is } D^{*0}\\
&B_c^+ \to D_s^+\eta \to D^0K^+ \text{ exchange meson is } D^{*0}\\
&B_c^+ \to D_s^{*+}\rho^0 \to D^0K^+ \text{ exchange mesons are } D^0, D^{*0}\\
&B_c^+ \to D_s^{*+}\omega \to D^0K^+ \text{ exchange mesons are } D^0, D^{*0}\\
&B_c^+ \to D_s^+\pi^0 \to D^0K^+ \text{ exchange meson is } K^{*-}\\
&B_c^+ \to D_s^+\eta \to D^0K^+ \text{ exchange meson is } K^{*-}\\
&B_c^+ \to D_s^{*+}\rho^0 \to D^0K^+ \text{ exchange mesons are } K^-, K^{*-}\\
&B_c^+ \to D_s^{*+}\omega \to D^0K^+ \text{ exchange mesons are } K^-, K^{*-}.
\end{aligned}
\tag{15}
$$

The meson vertices correspond to FSI are seen in the figure 4. For example, consider the meson vertex ϕKK. This vertex shows the decay of ϕ meson into two KK mesons. The coupling constant of the vertex is obtained from the relation $g_{\phi KK} = (m_\phi / |\vec{p_K}|)\sqrt{(6\pi\Gamma^{\text{exp}}_{\phi\to KK}) / |\vec{p_K}|}$ in which in the particles data group (PDG), $\Gamma^{\text{exp}}_{\phi\to KK}$ is given by 2.09 MeV [9]. The parameter $|\vec{p}|$ is the momentum of the K-meson in the rest frame of the ϕ-meson. The rest of the coupling constant of the other meson vertices, can be obtained from the similar relation for ϕ-meson decay. The vertex factors which include a vector meson V and two pseudo scalar mesons P, can be obtained from $\langle P_1(p_1)P_2(p_2)|i\ell|V_3(\epsilon_3, p_3)\rangle = -ig_{p_1p_2V}\epsilon_3\cdot(p_1+p_2)$. As an example, vertex factor of $J/\psi DD$ and ϕKK are:

$$\langle D(p_1)D(p_2)|i\ell|J/\psi(\epsilon_3, p_3)\rangle = -ig_{J/\psi DD}\epsilon_3\cdot(p_1+p_2),$$
$$\langle K(p_1)K(p_2)|i\ell|\phi(\epsilon_3, p_3)\rangle = -ig_{\phi KK}\epsilon_3\cdot(p_1+p_2). \quad (16)$$

The factor of the $\langle P_1(p_1)V_2(\epsilon_2, p_2)|i\ell|V_3(\epsilon_3, p_3)\rangle = -i\sqrt{2}g_{p_1V_2V_3}\epsilon_{\mu\nu\alpha\beta}\epsilon_2^{\mu}\epsilon_3^{*\nu}p_1^{\alpha}p_2^{\beta}$ is used for the vertex factors which include a pseudo scalar meson P and two vector mesons V, so, the vertex factor of $J/\psi D^*D$ and ϕK^*K are:

$$\langle D(p_1)D^*(\epsilon_2, p_2)|i\ell|J/\psi(\epsilon_3, p_3)\rangle = -i\sqrt{2}g_{J/\psi D^*D}\epsilon_{\mu\nu\alpha\beta}\epsilon_2^{\mu}\epsilon_3^{*\nu}p_1^{\alpha}p_2^{\beta}$$
$$\langle K(p_1)K^*(\epsilon_2, p_2)|i\ell|\phi(\epsilon_3, p_3)\rangle = -i\sqrt{2}g_{\phi K^*K}\epsilon_{\mu\nu\alpha\beta}\epsilon_2^{\mu}\epsilon_3^{*\nu}p_1^{\alpha}p_2^{\beta}. \quad (17)$$

Finally, one can calculate the amplitude of graphs that have meson loops as shown in figure 4, for the case in which both mesons are pseudo scalar as:

$$\begin{aligned} &M(B_c^+(p_{B_c^+}) \to P_1(p_1)P_2(p_2) \to P_3(p_3)P_4(p_4)) \\ &= \frac{1}{2}\int \frac{d^3\vec{p_1}}{2E_1(2\pi)^3}\frac{d^3\vec{p_2}}{2E_2(2\pi)^3} \\ &\times (2\pi)^4\delta^4(p_{B_c^+} - p_1 - p_2) \\ &\times M(B_c^+ \to P_1P_2)G(P_1P_2 \to P_3P_4), \end{aligned} \quad (18)$$

in equation (18), P_1P_2 and P_3P_4 mesons are intermediate and final state mesons, respectively. Also, $G(P_1P_2 \to P_3P_4)$ shows meson vortices factors which include the product of top factor in down factor in each graph. For the case which both intermediate mesons are vector mesons, the amplitude of graph, including meson loop, is obtained from the following equation:

$$\begin{aligned} &M(B_c^+(p_{B_c^+}) \to V_1(\epsilon_1, p_1)V_2(\epsilon_2, p_2) \to P_3(p_3)P_4(p_4)) \\ &= \frac{1}{2}\int \frac{d^3\vec{p_1}}{2E_1(2\pi)^3}\frac{d^3\vec{p_2}}{2E_2(2\pi)^3} \\ &\times (2\pi)^4\delta^4(p_{B_c^+} - p_1 - p_2)f_{V_1}m_{V_1}V_{CKM} \\ &\times \Bigg[(\epsilon_1^*\cdot\epsilon_2^*)(m_{B_c^+} + m_2)A_1^{B_c^+\to V_2}(m_1^2) \\ &- (\epsilon_1^*\cdot p_{B_c^+})(\epsilon_2^*\cdot p_{B_c^+})\frac{2A_2^{B_c^+\to V_2}(m_1^2)}{m_{B_c^+}+m_2}\Bigg]G(P_1P_2 \to P_3P_4), \end{aligned} \quad (19)$$

which is assumed that, the vector mesons V_2 and V_1 are set in form factor and vacuum state, respectively. But, in the case when vector mesons V_1 and V_2 are setted in form factor and vacuum state, respectively, indices are replaced in equation (19). So, the first amplitude of the nineteen amplitudes of figure 4. i.e amplitude $B_c^+ \to D^0K^+$, with $D^0(q)$ as an exchanged meson, can be written as:

$$\begin{aligned} M(4a, D^0) &= i\frac{G_F}{4\sqrt{2}\pi m_{B_c^+}}f_{D_s^{*+}}m_{D_s^{*+}}a_1V_{cb}^* \\ &\times V_{cs}g_{J/\psi DD}g_{D_s^*DK}\int_{-1}^{+1}|\vec{p_1}|d(\cos\theta)\frac{F^2(q^2, m_D^2)}{q^2 - m_D^2} \\ &\times \Bigg[(m_{B_c^+} + m_{J/\psi})A_1^{B_c^+\to J/\psi}(m_{D_s^{*+}}^2)K_1 \\ &- \frac{2A_2^{B_c^+\to J/\psi}(m_{D_s^{*+}}^2)}{m_{B_c^+} + m_{J/\psi}}K_1'\Bigg], \end{aligned} \quad (20)$$

in equation (20) θ is the angle between momentums $\vec{p_1}$ and $\vec{p_3}$, also $q = p_1 - p_3 = p_4 - p_2$ is the momentum of exchanged meson. In this case, $q^2 - m_D^2$ has the form $m_1^2 - 2E_1E_3 + 2|\vec{p_1}||\vec{p_3}|\cos\theta$. The parameters K_1 and K_1' show the product of polarization vectors of vector mesons where $K_1 = (\epsilon_1\cdot p_3)(\epsilon_2\cdot p_4)(\epsilon_1^*\cdot\epsilon_2^*)$ and $K_1' = (\epsilon_1\cdot p_3)(\epsilon_2\cdot p_4)(\epsilon_1\cdot p_2)(\epsilon_2\cdot p_1)$. The second amplitude shows the same process as before with the difference that, the exchanged meson in this case is vector meson, so, we have:

$$\begin{aligned} M(4a, D^{*0}) &= i\frac{G_F}{8\sqrt{2}\pi m_{B_c^+}}f_{D_s^{*+}}m_{D_s^{*+}}a_1V_{cb}^* \\ &\times V_{cs}g_{J/\psi D^*D}g_{D_s^*D^*K}\int_{-1}^{+1}|\vec{p_1}|d(\cos\theta)\frac{F^2(q^2, m_{D^*}^2)}{q^2 - m_{D^*}^2} \\ &\times \Bigg[(m_{B_c^+} + m_{J/\psi})A_1^{B_c^+\to J/\psi}(m_{D_s^{*+}}^2)K_2 \\ &- \frac{2A_2^{B_c^+\to J/\psi}(m_{D_s^{*+}}^2)}{m_{B_c^+} + m_{J/\psi}}K_2'\Bigg], \end{aligned} \quad (21)$$

where $K_2 = \epsilon_{\mu\nu\alpha\beta}\epsilon_1^{\mu}\epsilon_{D^*}^{\nu}p_3^{\alpha}p_1^{\beta}\epsilon_{\rho\sigma\lambda\eta}\epsilon_2^{\rho}\epsilon_{D^*}^{\sigma}p_4^{\lambda}p_2^{\eta}(\epsilon_1^*\cdot\epsilon_2^*)$ and $K_2' = \epsilon_{\mu\nu\alpha\beta}\epsilon_1^{\mu}\epsilon_{D^*}^{\nu}p_3^{\alpha}p_1^{\beta}\epsilon_{\rho\sigma\lambda\eta}\epsilon_2^{\rho}\epsilon_{D^*}^{\sigma}p_4^{\lambda}p_2^{\eta}(\epsilon_1\cdot p_{B_c})(\epsilon_2\cdot p_{B_c})$. In this amplitude, $q^2 - m_{D^*}^2$ has the form of $m_1^2 + m_3^2 - m_{D^*}^2 - 2E_1E_3 + 2|\vec{p_1}||\vec{p_3}|\cos\theta$.

$$\begin{aligned} M(4b, K^-) &= i\frac{G_F}{4\sqrt{2}\pi m_{B_c^+}}f_{\phi}m_{\phi}a_4V_{tb}^*V_{ts}g_{D_s^*DK}g_{\phi KK} \\ &\times \int_{-1}^{+1}|\vec{p_1}|d(\cos\theta)\frac{F^2(q^2, m_K^2)}{q^2 - m_K^2} \\ &\times \Bigg[(m_{B_c^+} + m_{D_s^{*+}})A_1^{B_c^+\to D_s^{*+}}(m_{\phi}^2)K_1 \\ &- \frac{2A_2^{B_c^+\to D_s^{*+}}(m_{\phi}^2)}{m_{B_c^+} + m_{D_s^{*+}}}K_1'\Bigg]. \end{aligned} \quad (22)$$

To calculate the amplitude $M(4b, K^{*-})$ it is enough to do as follow:

Table 1. Input parameters.

I) Mesons masses decay constants (in units of MeV) [9]		
$m_{B_c^+} = 6274.9 \pm 0.08$	$m_{J/\psi} = 3096.900 \pm 0.006$	$m_{D_s^{+*}} = 2112.2 \pm 0.4$
$m_{D_s^+} = 1968.34 \pm 0.07$	$m_{D^0} = 1864.83 \pm 0.05$	$m_\phi = 1019.461 \pm 0.016$
$m_\omega = 782.65 \pm 0.12$	$m_{\rho^0} = 775.26 \pm 0.25$	$m_\eta = 547.862 \pm 0.017$
$m_{K^0} = 497.611 \pm 0.013$	$m_{K^+} = 493.677 \pm 0.016$	$m_{\pi^0} = 134.9773 \pm 0.0005$
$f_{B_c} = 489 \pm 4$	$f_{J/\psi} = 418 \pm 9$	$f_{D_s^*} = 315 \pm 8$
$f_{D_s} = 294 \pm 7$	$f_D = 234 \pm 15$	$f_\phi = 215 \pm 5$
$f_\rho = 210 \pm 4$	$f_\omega = 195 \pm 2$	$f_K = 159.8 \pm 1.84$
$f_\pi = 130.70 \pm 0.46$	$f_\eta = 63.6 \pm 0.23$	
II) CKM matrix elements [9]		
$V_{ub} = 0.00394 \pm 0.00036$	$V_{cb} = 0.0422 \pm 0.0008$	$V_{tb} = 1.019 \pm 0.025$
$V_{us} = 0.2243 \pm 0.0005$	$V_{cs} = 0.997 \pm 0.017$	$V_{ts} = 0.0394 \pm 0.0023$
III) Coupling constants		
$g_{DD\rho} = 2.52$	$g_{D^*D\rho} = 2.82$	$g_{J/\psi DD} = 7.71$
$g_{J/\psi D^*D} = 8.64$ [24]	$g_{D^*D\pi} = 12.5$ [25]	$g_{D_s^*DK} = 2.89 \pm 0.25$ [26]
$g_{D_s^*D^*K} = 9.23$	$g_{K^*K\pi} = 4.6$ [27]	
$g_{K^*K\rho} = g_{K^*K\phi} = 6.48$	$g_{KK\rho} = g_{KK\phi} = 5.55$ [28]	
IV) Form factors [18, 29]		
$f_0^{B_c\to J/\psi}(m_{D_s}^2) = 0.57 \pm 0.21$	$f_+^{B_c\to J/\psi}(m_{D_s}^2) = 0.21 \pm 0.08$	$f_-^{B_c\to J/\psi}(m_{D_s}^2) = -0.75 \pm 0.23$
$f_0^{B_c\to J/\psi}(m_{D_s^*}^2) = 0.59 \pm 0.23$	$f_+^{B_c\to J/\psi}(m_{D_s^*}^2) = 0.22 \pm 0.08$	$f_-^{B_c\to J/\psi}(m_{D_s^*}^2) = -0.78 \pm 0.25$
$A_{1,2}^{B_c\to D_s^*}(m_{J/\psi}^2) = 0.27 \pm 0.04$	$A_1^{B_c\to D_s^*}(m_\phi^2) = 0.15 \pm 0.01$	$A_2^{B_c\to D_s^*}(m_\phi^2) = 0.13 \pm 0.01$
$F_0^{B_c\to D}(m_K^2) = 0.16 \pm 0.02$	$F_0^{B_c\to D_s}(m_K^2) = 0.28 \pm 0.02$	
$F_1^{B_c\to D_s}(m_{J/\psi}^2) = 0.58 \pm 0.07$	$F_1^{B_c\to D_s}(m_\phi^2) = 0.30 \pm 0.02$	
V) Wilson coefficients [15]		
$c_1 = 1.081$	$c_2 = -0.190$	$c_3 = 0.014$
$c_4 = -0.036$		

(1) convert 4 to 8 in the denominator of the first fraction line,
(2) replace $g_{D_s^*DK}\, g_{KK\phi}$ with $g_{D_s^*DK^*}g_{KK^*\phi}$,
(3) replace m_K with m_{K^*} in the face and denominator of the second fraction line,
(4) convert coefficients K_1 and K_1' to K_2 and K_2', respectively.

The FSI amplitude of the third graph of figure 4, the amplitude $B_c^+ \to D^+(p_1)K^0(p_2) \to D^0(p_3)K^+(p_4)$ with $\rho^-(q)$ as exchange meson can be written as:

$$M(4c) = -g_{DD\rho}g_{KK\rho} \times \int_{-1}^{+1} \frac{|\overrightarrow{p_1}|d(\cos\theta)}{16\pi m_{B_c^+}} M \times (B_c^+ \to D^+K^0) \frac{F^2(q^2, m_\rho^2)}{q^2 - m_\rho^2} K_3, \quad (23)$$

where $K_3 = p_{1\mu}\epsilon_\rho^\mu p_{2\nu}\epsilon_\rho^\nu$. The amplitude of the remaining graphs of figure 4 are obtained as written amplitudes. The dispersive part of the rescattering amplitude can be obtained from the absorptive parts via the dispersion relation [22, 23]

$$Dis4(m_{B_c^+}^2) = \frac{1}{\pi}\int_s^\infty \frac{M_{4a}(s') + M_{4b}(s') + M_{4c}(s') + \ldots + M_{4n}(s')}{s' - m_{B_c^+}^2} ds', \quad (24)$$

where s' is the square of the momentum carried by the exchanged particle and s is the threshold of intermediate states, in this case $s \sim m_{B_c^+}^2$. Finally, the total amplitude of the FSI for $B_c^+ \to D^0K^+$ decay is calculated as:

$$M(B_c^+ \to D^0K^+)_{FSI} = M(4a, D) + M(4a, D^*) + M(4b, K) + M(4b, K^*) + M(4c) + M(4d, \pi) + M(4d, \rho) \ldots M(4l, K) + M(4l, K^*) + Dis4. \quad (25)$$

At the end, one can calculate the branching ratio as:

$$\mathcal{B}(B_c^+ \to D^0K^+) = \frac{1}{\Gamma_{\text{tot}}} \frac{|M(B_c^+ \to D^0K^+)|^2}{16\pi m_{B_c^+}}, \quad (26)$$

where $\Gamma_{\text{tot}} = 1.282 \times 10^{-12}$ GeV.

Table 2. The branching ratio of $B_c^+ \to D^0K^+$ decay with $f_c/f_u = [0.4, 1.2]\%$, $\eta = 1 \sim 3$ and experimental data (BR in EXP is the branch ratio present in the experimental observation).

Contributions	η	$\frac{f_c}{f_u} \times \mathcal{B}(B_c^+ \to D^0K^+)(\times 10^{-7})$	$\mathcal{B}(B_c^+ \to D^0K^+)$ $(\times 10^{-5})$
QCDF	—	$(2.56 \pm 0.71)\% \sim (7.69 \pm 2.12)\%$	$(6.41 \pm 1.77)\%$
	1.0	$(0.70 \pm 0.12) \sim (2.10 \pm 0.35)$	1.75 ± 0.29
FSI	2.0	$(3.30 \pm 0.52) \sim (9.90 \pm 1.55)$	8.25 ± 1.29
	3.0	$(6.06 \pm 0.90) \sim (18.18 \pm 2.70)$	15.15 ± 2.25
EXP [6]	—	9.30 ± 0.60	—
BR in EXP	—	—	$(3.72 \pm 0.24) \sim (11.16 \pm 0.72)$

4. Numerical results

The meson masses and decay constants needed in our calculations, are listed in the section 1 in the table 1. The Cabibbo-Kobayashi-Maskawa (CKM) matrix is a 3×3 unitary matrix, the elements of this matrix can be parameterized by three mixing angles A, λ and ρ and a CP-violating phase η [9]: $V_{us} = \lambda$, $V_{ub} = A\lambda^3(\rho - i\eta)$, $V_{cs} = 1 - \lambda^2/2$, $V_{cb} = A\lambda^2$, $V_{ts} = -A\lambda^2$ and $V_{tb} = 1$, the results are shown in section 2 in the table 1. The values of the coupling constants and the corresponding form factors are given in sections 3 and 4 in the table 1, respectively. The Wilson coefficients c_1, c_2, c_3 and c_4 in the effective weak Hamiltonian have been reliably evaluated by the next-to-leading logarithmic order. To proceed, we use the following numerical values at $\mu = m_b$ scale, which have been obtained in the NDR scheme. These coefficient numbers are inserted in the section 5 of the table 1 [15]. Using the parameters relevant for the $B_c^+ \to D^0K^+$ decay, we get flavor averaged branching ratio for the QCD factorization method as:

$$\mathcal{B}(B_c^+ \to D^0K^+)_{QCDF} = (6.41 \pm 1.77) \times 10^{-7}. \quad (27)$$

Now, applying the effects of the FSI, we obtain the branching ratios of $B_c^+ \to D^0K^+$ decay with different values of η, the results are shown in the table 2. In the first column of table 2, the experimental result for $(f_c/f_u)\mathcal{B}(B_c^+ \to D^0K^+)$ is $(9.30 \pm 0.60) \times 10^{-7}$ and the result of current calculations by using the QCDF approach is from $((2.56 \pm 0.71) \times 10^{-9})_{f_c/f_u=0.4\%}$ to $((7.69 \pm 2.12) \times 10^{-9})_{f_c/f_u=1.2\%}$ that is about 10^2 times smaller than experimental one. By entering effects of FSI and select $\eta = 1$, the result of previous calculations (QCDF approach) is improved by 27 times and reaches $((0.70 \pm 0.12) \times 10^{-7})_{f_c/f_u=0.4\%}$ to $((2.10 \pm 0.35) \times 10^{-7})_{f_c/f_u=1.2\%}$ in which the upper limit ($f_c/f_u = 1.2\%$ case) is still 4 times smaller the experience. By choosing $\eta = 2$ the situation becomes much better and the value $((3.30 \pm 0.52) \times 10^{-7})_{f_c/f_u=0.4\%}$ to $((9.90 \pm 1.55) \times 10^{-7})_{f_c/f_u=1.2\%}$ is obtained. In this case ($\eta = 2$ and $f_c/f_u = 1.2\%$) the upper limit is compatible with the experimental result. Choosing $\eta = 3$ makes the value of the lower limit to have good compatibility with the experimental result. There is a similar description to the second column of the table 2. Note that, considering $f_c/f_u = 1.2\%$ and in addition to the conventional selection of η, we choose its value according to the mass of the exchange meson, as $\eta = 3$ for exchange meson of D^* (or D) and $\eta = 1$ for exchange meson of K^* (or K). The result is as follow:

$$\frac{f_c}{f_u} \times \mathcal{B}(B_c^+ \to D^0K^+)_{FSI} = (11.67 \pm 1.72) \times 10^{-7}. \quad (28)$$

5. Conclusion

The decay of $B_c^+ \to D^0K^+$ has been observed by the LHCb collaboration with the measurement of the branching fraction multiplied by the production rates for B_c^+ relative to B^+ mesons as $(f_c/f_u)\mathcal{B}(B_c^+ \to D^0K^+)_{EXP} = (9.30 \pm 0.60) \times 10^{-7}$. In this paper we have calculated the branching ratio of the $B_c^+ \to D^0K^+$ decay using the QCDF theorem. The numerical value of this calculation is $\mathcal{B}(B_c^+ \to D^0K^+)_{QCDF} = (6.41 \pm 1.77) \times 10^{-7}$. For direct comparison with the experimental result we have multiplied this value by $f_c/f_u = [0.4, 1.2]\%$, the result of this calculation is $(f_c/f_u)\mathcal{B}(B_c^+ \to D^0K^+)_{QCDF}$ =$(2.56 \pm 0.71) \times 10^{-9} \sim (7.69 \pm 2.12) \times 10^{-9}$ that is about 10^2 times smaller than experimental one.

On the other hand, to obtain the branch ratio value that exists within the experimental observation (INEXP), we have divided the experimental value $((9.30 \pm 0.60) \times 10^{-7})$ by $f_c/f_u = [0.4, 1.2]\%$, the result is $\mathcal{B}(B_c^+ \to D^0K^+)_{INEXP} = (3.72 \pm 0.24) \times 10^{-5} \sim (11.16 \pm 0.72) \times 10^{-5}$, this value is also 10^2 times larger than our predicted value using the QCDF approach. Therefore, we have tried to use a method that covers the entire experimental range. For achieve this goal we have applied the FSI effects, in which such decays are highly dependent on the phenomenology parameter which appear in the form factors of the long distance distributions. By applying these effects and considering that the decay in this work is done only through the cross section processes, four intermediate states have been created. The contribution of all these intermediate decays have been included in the final amplitude. Finally, we have calculated the branching ratio by using the FSI effects for various values of the phenomenological parameter, $\eta = 1 \sim 3$. The obtained results have been covered the branch ratio that exists inside the experimental view. In another selection of η, we have fixed $\eta = 1$ and $\eta = 3$ for light and heavy intermediate mesons, respectively, with this selection and fixing f_c/f_u with 1.2% more acceptable result has been obtained.

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