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The branching fraction calculations of $B_c^+ \to \psi(2S)\pi^+$, $B_c^+ \to J/\psi K^+$ and $B_c^+ \to J/\psi D_s^+$ decays relative to that of the $B_c^+ \to J/\psi\pi^+$ mode

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The weak decay of B_c^+ into $\psi(2S)\pi^+$, $J/\psi K^+$ and $J/\psi D_s^+$ mesons, observed by LHCb collaboration for the first time, are calculated in the model which takes into account the "factorizable" contributions and "nonfactorizable" corrections. The decays of B_c^+ mesons into charmonia and light hadrons are expected to be well described by the factorization approximation. In the standard model, $B_c^+ \to \psi(2S)\pi^+$, $J/\psi K^+$ decays occur through only the tree-level diagrams and so there are no CP violation in these channels. The decay $B_c^+ \to \psi(2S)\pi^+$ is expected to proceed mainly via a $\bar{b} \to \bar{c}u\bar{d}$ transition because the $B_c^+ \to J/\psi\pi^+$ decay has identical final state and similar event topology, where it is chosen as the relative branching fraction channel. The ratio of branching fractions $\mathcal{B}(B_c^+ \to J/\psi K^+)/\mathcal{B}(B_c^+ \to J/\psi\pi^+)$ is of particular interest since the CKM matrix element is suppressed by a factor $|V_{us}/V_{ud}|^2 \sim 0.05$, in which the $B_c^+ \to J/\psi K^+$ occur through $\bar{b} \to \bar{c}u\bar{s}$ transition, but the dominant amplitude of the decay $B_c^+ \to J/\psi\pi^+$ is a $\bar{b} \to \bar{c}u\bar{d}$ transition. The decay $B_c^+ \to J/\psi D_s^+$ is examined by color-allowed, color-suppressed spectator and weak annihilation diagrams. The weak annihilation topology, in contrast to decays of other beauty hadrons, is not suppressed and can contribute significantly to the decay amplitude. Because of the $B_c^+ \to \psi(2S)\pi^+$, $B_c^+ \to J/\psi K^+$ and $B_c^+ \to J/\psi D_s^+$ branching fractions are calculated relative to the $B_c^+ \to J/\psi\pi^+$ decay, this decay mode is estimated separately, the ratio between them are 0.327 ± 0.028, 0.074 ± 0.0057 and 3.257 ± 0.293, respectively, that are compatible with the experimental data.

Keywords: B_c^+ meson decays; factorizable and nonfactorizable contributions; factorization; branching fractions.

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1. Introduction

B_c meson is one of the most interesting mesons that can be studied at the Tevatron and LHC, the discovery of the B_c was reported by the CDF collaboration in the

$B_c \to J/\psi l^{\pm}\bar{\nu}_l$ process at Fermilab.[1] After that, the decay mode $B_c^{\pm} \to J/\psi\pi^{\pm}$ has been observed by CDF and D0 collaboration significance of more than 8σ and 5σ, respectively.[2,3] Studies of B_c properties are important because it is made of two different heavy quarks, bottom-charm antiquark–quark pair. Each of the quarks can participate in a weak interaction in which other quark participates as a spectator. For this, B_c is also the only meson in which decays of both heavy quarks compete with each other, therefore a wide range of decay channels are possible. However, a significant number of these channels has not been observed yet.[4] Unlike B^0, B^+ and B_s^0 mesons, more than 70% of the B_c^+ width is due to c-quark decays, in which $c \to s$ transition has been observed with $B_c^+ \to B_s^0\pi^+$ decays.[5] Around 20% of its width is due to the b-quark decays.[6] In charmless final states, the $\bar{b}c \to W^+ \to \bar{q}q$ annihilation amplitudes account for only 10% of the B_c^+ width.[7]

In this research, the $B_c^+ \to \psi(2S)\pi^+$, $B_c^+ \to J/\psi K^+$ and $B_c^+ \to J/\psi D_s^+$ branching fractions are calculated relative to the $B_c^+ \to J/\psi\pi^+$ one, these analyses are studied using a simple model based on the framework of the factorization approach.[8] The decay $B_c^+ \to \psi(2S)\pi^+$ with $\psi(2S) \to \mu^+\mu^-$ has been observed by the LHCb experiment for the first time. The branching fraction of $B_c^+ \to \psi(2S)\pi^+$ decay relative to that of the $B_c^+ \to J/\psi\pi^+$ mode has been measured by them is $R_{\psi(2S)/J/\psi} = \mathcal{B}(B_c^+ \to \psi(2S)\pi^+)/\mathcal{B}(B_c^+ \to J/\psi\pi^+) = 0.250 \pm 0.068(\text{stat}) \pm 0.014(\text{syst})$,[4] after that they have obtained $R_{\psi(2S)/J/\psi} = 0.268 \pm 0.032(\text{stat}) \pm 0.007(\text{syst})$ at center-of-mass energies $\sqrt{s} = 7$ and 8 TeV.[9]

The LHCb collaboration have also observed the $B_c^+ \to J/\psi K^+$ decay for the first time using a data sample, corresponding to an integrated luminosity of 1 fb^{-1}, collected by them in pp collisions at a center-of-mass energy of 7 TeV. The ratio of the branching fraction of $B_c^+ \to J/\psi K^+$ to that of $B_c^+ \to J/\psi\pi^+$ was measured to be $R_{K/\pi} = \mathcal{B}(B_c^+ \to J/\psi K^+)/\mathcal{B}(B_c^+ \to J/\psi\pi^+) = 0.069 \pm 0.019 \pm 0.005$.[10] The uncertainty on this value is too large to discriminate between the predictions quoted above. They have continued their analysis with an additional sample taken at a center-of-mass energy of 8 TeV corresponding to an integrated luminosity of 2 fb^{-1}. Owing to improvements in the analysis method as well as the increase in the data sample size, the statistical uncertainty is reduced by a factor of more than two, the systematic uncertainty is also reduced. With this remeasurement, they have obtained $R_{K/\pi} = 0.079 \pm 0.007 \pm 0.003$.[11]

The decay $B_c^+ \to J/\psi D_s^+$ has been observed for the first time using a dataset, corresponding to an integrated luminosity of 3 fb^{-1}, collected by the LHCb experiment in proton–proton collisions at center-of-mass energies of $\sqrt{s} = 7$ and 8 TeV. The following ratios of branching fractions have been measured for $B_c^+ \to J/\psi D_s^+$ mode: $\mathcal{B}(B_c^+ \to J/\psi D_s^+)/\mathcal{B}(B_c^+ \to J/\psi\pi^+) = 2.90 \pm 0.57(\text{stat}) \pm 0.24(\text{syst})$.[12]

In the study of the leading Feynman diagrams, a color-allowed spectator diagram, only possible diagram (in which is the dominant coefficient of a_1 contribution), is considered to the $B_c^+ \to \psi(2S)\pi^+$, $B_c^+ \to J/\psi K^+$ and $B_c^+ \to J/\psi\pi^+$ decays. All the "nonfactorizable" effects (coming from the vertex-correction and hard spectator-scattering diagrams) are encoded in the coefficients a_1, which are process dependent and can be obtained by perturbative calculation. For the $B_c^+ \to J/\psi D_s^+$ decay, in addition to the mentioned diagram, a color-suppressed spectator (coefficient of a_2 contribution) and a weak annihilation (coefficient of b_2 contribution) diagrams are applied to the decay amplitude, in this decay mode, the weak annihilation topology, unlike other B meson decays, is not suppressed and can contribute to the amplitude of the $B_c^+ \to J/\psi D_s^+$ decay. The ratios of branching fractions depend on the ratios of the form factors, values of the three-momentum of the final mesons, decay constants, CKM matrix elements and Wilson coefficients. For example, the ratios of branching fraction $\mathcal{B}(B_c^+ \to J/\psi K^+)/\mathcal{B}(B_c^+ \to J/\psi\pi^+)$ become:

$$(|\mathbf{p}_K|/|\mathbf{p}_\pi|)\left|\left[V_{us} f_K A_0^{B_c^+\to J/\psi}(m_K^2) a_1(J/\psi K)\right]\Big/\left[V_{ud} f_\pi A_0^{B_c^+\to J/\psi}(m_\pi^2) a_1(J/\psi\pi)\right]\right|^2.$$

By entering the "factorizable" contributions and "nonfactorizable" corrections into calculations, the ratios of branching fractions become:

$$\begin{aligned} \frac{\mathcal{B}(B_c^+ \to \psi(2S)\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} &= 0.327 \pm 0.028\,, \\ \frac{\mathcal{B}(B_c^+ \to J/\psi K^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} &= 0.074 \pm 0.0057\,, \\ \frac{\mathcal{B}(B_c^+ \to J/\psi D_s^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} &= 3.257 \pm 0.293\,, \end{aligned} \tag{1}$$

which are compatible with the experimental data $0.268 \pm 0.032 \pm 0.007$, $0.079 \pm 0.007 \pm 0.003$ and $2.900 \pm 0.570 \pm 0.240$, respectively.[9,11,12]

2. Decay Amplitudes

2.1. *Amplitude of the $B_c^+ \to \psi(2S)\pi^+$, $J/\psi K^+$ and $J/\psi\pi^+$ decays including the "factorizable" and "nonfactorizable" effects*

In the following, we calculate the amplitude of the $B_c^+ \to \psi M^+$ decay as in Ref. 13, where ψ denotes $\psi(2S)$ and J/ψ and M denotes K^+ and π^+. Different from $B_{u,d,s}$ mesons, the B_c^+ system consists of two heavy quarks $\bar{b}$ and c, which can decay individually. Here, we will consider $\bar{b}$ decays while c acts as a spectator. According to Fig. 1, at the quark level, $B_c^+ \to \psi(2S)\pi^+$ and $B_c^+ \to J/\psi K^+(\pi^+)$ decays are characterized by $\bar{b} \to [\bar{c}u\bar{d}(\bar{s})]$ transition and the corresponding effective Hamiltonian is given by $H_{\rm eff} = (G_F/\sqrt{2})\sum_{p=u,c}\lambda_p(c_1 Q_1^p + c_2 Q_2^p + \sum_{i=3,\ldots,10} c_i Q_i + c_{7\gamma} q_{7\gamma} + c_{8g} Q_{8g})$ where λ_p is the CKM matrix elements, c_i are the Wilson coefficients evaluated at the renormalization scale μ, $Q_{1,2}^p$ are the left-handed current–current operators arising from W-boson exchange, $Q_{3,\ldots,6}$ and $Q_{7,\ldots,10}$ are QCD and electroweak penguin operators, and $Q_{7\gamma}$ and Q_{8g} are the electromagnetic and chromomagnetic dipole operators. Because only the tree-level diagram exist for this decay mode, we have considered current–current operators Q_1^c and Q_2^c as: $Q_1^c = (\bar{b}_\alpha c_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A}$ and $Q_2^c = (\bar{b}_\alpha u_\beta)_{V-A}(\bar{d}_\beta c_\alpha)_{V-A}$. Here, α and β are

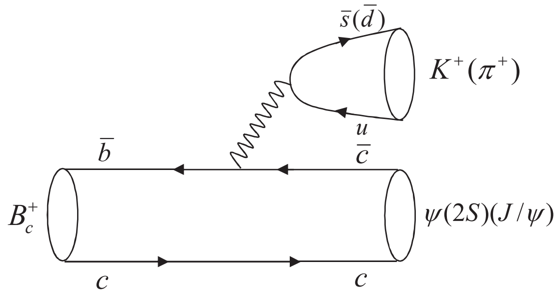

Fig. 1. Feynman diagram for $B_c^+ \to \psi(2S)\pi^+$ and $B_c^+ \to J/\psi K^+(\pi^+)$ decays.

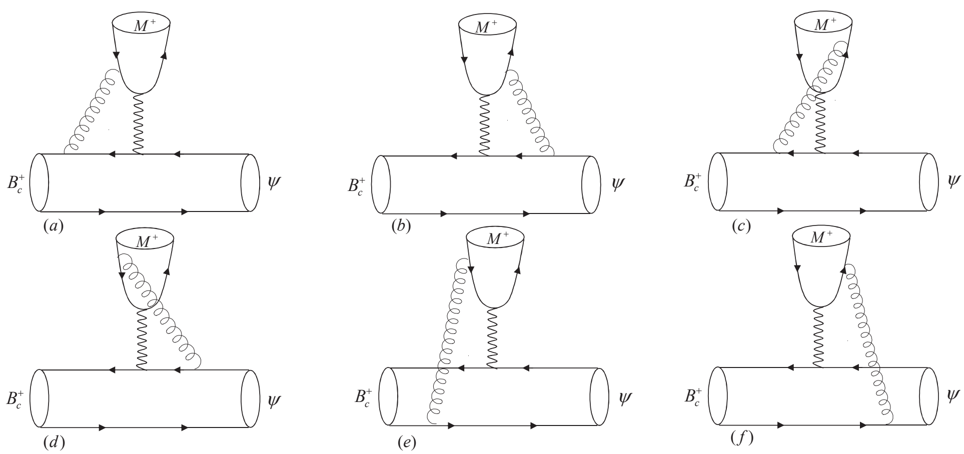

Fig. 2. Nonfactorizable diagrams for $B_c^+ \to \psi M^+$ decay.

the SU(3) color indices and the subscript $V - A$ represent the chiral projection $1 - \gamma_5$. The amplitude of the $B_c^+ \to \psi M^+$ decay by using the color-allowed external W-emission tree diagram becomes: $(G_F/\sqrt{2})V_{cb}V_{ud}^*\langle\psi M^+|(c_1Q_1^c + c_2Q_2^c)|B_c^+\rangle_F$, where $\langle\psi M^+|(c_iQ_i^c|B_c^+\rangle_F$ is the factorized hadronic matrix element, which has the same definition as that in the "nonfactorizable" approach. All the "nonfactorizable" effects are encoded in the coefficient a_1, which are process-dependent and can be obtained perturbatively by calculating the diagrams in Fig. 2. This coefficient is split into two terms: the first term contains the naive factorization contribution and the vertex correction, (a)–(d) panels of Fig. 2. While the second one arises from the hard spectator interactions, (e) and (f) panels of Fig. 2. The hard-scattering interactions give leading twist and chirally-enhanced twist-3 contributions to the kernels effects. We include these hard-scattering contributions as parts of the coefficient a_1, although they are not related to factorized matrix elements in the usual sense. Only the twist-2 terms are dominated by hard gluon exchange and thus calculable. The general form of the coefficient a_1 at next-to-leading order in α_s can be written as $a_1(\psi M^+) = (c_1 + c_2/3)N(M^+) + (\alpha_s/9\pi)c_2[V(M^+) + (4\pi^2/3)H(\psi M^+)]$ (see Refs. 14–16 for further explanation), where $N(M^+)$ is the leading-order coefficient which will be considered $N(M^+) = 1$. The quantities $V(M^+)$ account for one-loop vertex correction which is given by: $\int_0^1 dx\, 6x(1-x)F(x,z)$, the function $F(x,z)$ with $z = m_c/m_b$ can be written as $[3 + 2\ln(x/(1-x))]\ln z^2 - 7 + f(x,z) + f(1-x, 1/z)$, the contribution of $f(x,z)$ comes from the (a) and (c) diagrams in Fig. 2 with the gluon coupling to the b quark, whereas $f(1-x, 1/z)$ arises from the (b) and (d) diagrams with the gluon coupling to the charm quark. The function $f(x,z)$ is given by

$$f(x,z) = -\frac{x(1-z^2)[3(1-x(1-z^2)) + z]}{[1-x(1-z^2)]^2}\ln[x(1-z^2)] - \frac{z}{1-x(1-z^2)} + 2\left[\frac{\ln[x(1-z^2)]}{1-x(1-z^2)} - \ln^2[x(1-z^2)] - \mathrm{Li}_2[1-x(1-z^2)] - [x \to (1-x)]\right], \tag{2}$$

where $\mathrm{Li}_2 = -\int_0^x dt(1/t)\ln(1-t)$ is the dilogarithm. Note that the terms in the large square brackets in the definition of the function $f(x,z)$ are odd under the exchange $x \leftrightarrow (1-x)$ and thus vanish for a symmetric light-cone distribution amplitude. The imaginary part of the $F(x,z)$ that arises from $f(1-x, 1/z)$ can be obtained by recalling that z^2 is $z^2 - i\epsilon$ with $\epsilon > 0$ infinitesimal, as

$$\mathrm{Im}\, F(x,z) = -\pi\frac{(1-x)(1-z^2)[3(1-x(1-z^2)) + z]}{[1-x(1-z^2)]^2} - \pi\left[\ln[1-x(1-z^2)] + 2\ln x + \frac{z^2}{1-x(1-z^2)} - [x \to (1-x)]\right]. \tag{3}$$

The correction from hard gluon exchange between M^+ and the spectator quark, $H(\psi M^+)$, is given by $\left(f_{B_c^+} f_{M^+} \Big/ \left(m^2_{B_c^+} A_0^{B_c^+\to\psi}\right)\right) \int_0^1 d\xi\, \Phi_{B_c^+}(\xi)/\xi \int_0^1 dx\, \Phi_{M^+}(x)/ (1-x)\int_0^1 dy\, \Phi_\psi^L(y)/(1-y(1-z^2))$. The hard spectator scattering contribution depends on the wave function $\Phi_{B_c^+}(\xi)$ through the integral $\int_0^1 d\xi\, \Phi_{B_c^+}(\xi)/\xi = m_{B_c^+}/\Lambda_{\mathrm{QCD}}$. For coefficient of a_1, we obtain $a_1(z \neq 0) = 1.0353 + 0.0024i$ by considering the mass of the quark c and in the case of ignoring the m_c against the m_b we get $a_1(z = 0) = 1.0307 - 0.0139i$, it is easily seen that $|a_1(z \neq 0)|/ |a_1(z = 0)| = 1.0043$, ignoring the mass of the c quark will not be affected in calculation. For $z \to 0$, we then find

$$V(M^+) = \int_0^1 dx\, 6x(1-x)[12\ln(m_b/\mu) + 3(1-2x)/(1-x)\ln(x) - 3i\pi - 18] \tag{4}$$

and

$$H(\psi M^+) = \frac{f_{B_c^+} f_{M^+}}{m^2_{B_c^+} A_0^{B_c^+\to\psi}} \int_0^1 d\xi \frac{\Phi_{B_c^+}(\xi)}{\xi} \int_0^1 dx \frac{\Phi_{M^+}(x)}{1-x} \int_0^1 dy \frac{\Phi_\psi^L(y)}{1-y}\,. \tag{5}$$

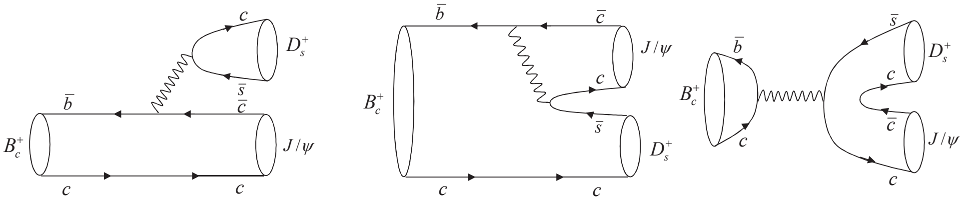

Fig. 3. Feynman diagrams for $B_c^+ \to J/\psi D_s^+$ decay.

In this decay mode, the ψ meson is placed in the form factor, the meson of M^+ is produced from the vacuum state, therefore the amplitude of this decay consist of $\langle B_c^+ \to \psi\rangle$ multiplied by $\langle 0 \to M^+\rangle$ which is "factorizable" term, where the form factor of $\langle B_c^+(p) \to \psi(\epsilon, q)\rangle$ is defined as:

$$\big[\big(m_{B_c^+} + m_\psi\big)\big(\epsilon_\mu - (\epsilon \cdot p)q'_\mu/q'^2\big)\big] A_1^{B_c^+\to\psi}(q'^2) - \big[(p+q)_\mu - \big(m^2_{B_c^+} - m^2_\psi\big)q'_\mu/q'^2\big]$$
$$\times (\epsilon \cdot p) A_2^{B_c^+\to\psi}(q'^2)/\big(m_{B_c^+} + m_\psi\big) + \big[2m_\psi(\epsilon \cdot p)q'_\mu/q'^2\big] A_0^{B_c^+\to\psi}(q'^2)\,,$$

where $q'_\mu = (p-q)_\mu$ and under the Lorentz condition $\epsilon \cdot q = 0$. In the rest frame of the decaying B_c^+ meson only longitudinally polarized ψ's is produced. $\epsilon \cdot p$ is then given by $m_{B_c^+}|\mathbf{p}_\psi|/m_\psi$. In this work, we make the extrapolation by using the formula in Ref. 17 $f(q'^2) = f(0)\exp(\sigma_1 q'^2 + \sigma_2 q'^4)$, where the $f(q'^2)$ denote the weak form-factors $A_0^{B_c^+\to\psi}(q'^2)$, $A_1^{B_c^+\to\psi}(q'^2)$ and $A_2^{B_c^+\to\psi}(q'^2)$ and σ_1, σ_2 and $f(0)$ are the parameters to be determined by the fitting procedure. For the decay constant of $\langle 0 \to M^+(p')\rangle$, we use $if_M p'^\mu$, with $p'^\mu = q'^\mu$. The expressions of decay amplitude for $B_c^+ \to \psi M^+$ within the factorization framework can be written as

$$A(B_c^+ \to \psi M^+) = \sqrt{2} G_F V_{cb} V^*_{ud} f_M A_0^{B_c^+\to\psi}(m_M^2) m_\psi (\epsilon_\psi \cdot p_M) a_1(\psi M^+)\,. \quad (6)$$

The decay rate of $B_c^+ \to \psi M^+$ in B_c^+ meson rest frame can be written as

$$\Gamma(B_c^+ \to \psi M^+) = \frac{1}{8\pi}\frac{|\mathbf{p}|}{m^2_{B_c^+}}|A(B_c^+ \to \psi M^+)|^2\,, \quad (7)$$

in which $|\mathbf{p}|$ is the absolute value of the three-momentum of the ψ or M^+ mesons that can be calculated via: $\sqrt{\big(m^2_{B_c^+} + m^2_\psi - m^2_M\big)^2 - 4m^2_{B_c^+} m^2_\psi}/\big(2m_{B_c^+}\big)$.

2.2. *Amplitude of the $B_c^+ \to J/\psi D_s^+$ decay*

Under the factorization approach and according to Fig. 3, a color-suppressed spectator (coefficient of a_2 contribution) and a weak annihilation (coefficient of b_2 contribution) diagrams are contributed in addition to the diagram a_1 described in the previous section, their contributions are added to the decay amplitude. The form factor of $\langle B_c^+(p) \to D_s^+(p')\rangle$ is parametrized as $\big\{\big[(p+p')_\mu - \big(m^2_{B_c^+} - m^2_{D_s^+}\big)q'_\mu/q'^2\big] F_1^{B_c^+\to D_s^+}(q'^2) + \big(m^2_{B_c^+} - m^2_{D_s^+}\big)q'_\mu/q'^2 F_0^{B_c^+\to D_s^+}(q'^2)\big\}$ and the decay constant of $\langle 0 \to J/\psi(q,\epsilon)\rangle$ is defined as $if_{J/\psi}m_{J/\psi}\epsilon^{\mu}_{J/\psi}$, here $q^{\mu} = q'^{\mu} = p^{\mu} - p'^{\mu}$. So the decay amplitude can be written as

$$A(B_c^+ \to J/\psi D_s^+) = \sqrt{2}G_F V_{cb}V_{cs}^* \Big\{ m_{J/\psi}\big(\epsilon_{J/\psi}\cdot p_{B_c^+}\big)\Big[f_{D_s^+}A_0^{B_c^+\to J/\psi}\Big(m^2_{D_s^+}\Big) \times a_1(J/\psi D_s^+) + f_{J/\psi}F_1^{B_c^+\to D_s^+}\big(m^2_{J/\psi}\big)a_2\big(J/\psi D_s^+\big)\Big] + \frac{1}{2}f_{B_c^+}f_{J/\psi}f_{D_s^+}b_2\Big\}, \quad (8)$$

where $F_1^{B_c^+\to D_s^+}\big(m^2_{J/\psi}\big) = F_1^{B_c^+\to D_s^+}(0)/\big(1 - m^2_{J/\psi}/m^2_{B_c^{+*}}\big)^2$.[18] In the annihilation topology, when all the basic building blocks equations are solved, for the case that one of the final state meson is vector and the other one is pseudoscalar, it is found that weak annihilation kernels exhibit endpoint divergence. Divergence terms are determined by $\int_0^1 dx/\bar{x}$ and $\int_0^1 dy/y$. For the liberation of the divergence, a small ϵ of $\Lambda_{\rm QCD}/m_B$ order was added to the denominator. So the answer to the integral becomes $\ln(1+\epsilon)/\epsilon$ form, which is shown with $X_A = (1+\rho_A e^{i\phi_A})\ln(m_B/\Lambda_{\rm QCD})$. Specifically, we treat X_A as an arbitrary parameter obtained by using $\rho_A = 0.5$ and a strong phase $\phi_A = -55°$.[15] The corresponding current–current annihilation b_2 coefficient becomes: $(8/9)\pi\alpha_s c_2\big[3(X_A - 4 + \pi^2/3) + r_\chi^{J/\psi} r_\chi^{D_s^+}(X_A^2 - 2X_A)\big]$. The light-cone expansion implies that only leading-twist distribution amplitudes are needed in the heavy-quark limit. There exist however a number of subleading quark–antiquark distribution amplitudes of twist 3, which have large normalization factors for pseudoscalar mesons, e.g. for the D_s^+, the ratio $r_\chi^{D_s^+}$ is defined as $\big(2m^2_{D_s^+}\big)/[(m_b - m_c)(m_s + m_c)]$ and for, J/ψ, vector meson, we have $r_\chi^{J/\psi} = (m_{J/\psi}/m_b)(f^{\perp}_{J/\psi}/f_{J/\psi})$.

3. Ratio of Branching Fractions and Numerical Results

The ratios $\mathcal{B}(B_c^+ \to \psi(2S)\pi^+)$, $\mathcal{B}(B_c^+ \to J/\psi K^+)$ and $\mathcal{B}(B_c^+ \to J/\psi D_s^+)$ to the $\mathcal{B}(B_c^+ \to J/\psi\pi^+)$ can now be estimated as follows:

$$\frac{\mathcal{B}(B_c^+ \to \psi(2S)\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} = \frac{|\mathbf{p}_{\psi(2S)}|}{|\mathbf{p}_{J/\psi}|}\left(\frac{A_0^{B_c^+\to\psi(2S)}(m_\pi^2)}{A_0^{B_c^+\to J/\psi}(m_\pi^2)}\right)^2\left(\frac{m_{\psi(2S)}}{m_{J/\psi}}\right)^2\left(\frac{\epsilon_{\psi(2S)}\cdot p_{B_c^+}}{\epsilon_{J/\psi}\cdot p_{B_c^+}}\right)^2,$$

$$\frac{\mathcal{B}(B_c^+ \to J/\psi K^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} = \left(\frac{|\mathbf{p}_K|}{|\mathbf{p}_\pi|}\right)^3\left(\frac{V_{us}}{V_{ud}}\right)^2\left(\frac{f_K}{f_\pi}\right)^2\left(\frac{A_0^{B_c^+\to J/\psi}(m_K^2)}{A_0^{B_c^+\to J/\psi}(m_\pi^2)}\right)^2\left(\frac{a_1(J/\psi K)}{a_1(J/\psi\pi)}\right)^2,$$

$$\frac{\mathcal{B}(B_c^+ \to J/\psi D_s^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} = \frac{|\mathbf{p}_{D_s^+}|}{|\mathbf{p}_{J/\psi}|}\left(\frac{V_{cs}}{V_{ud}}\right)^2\frac{1}{\Big[f_\pi m_{J/\psi}\big(\epsilon_{J/\psi}\cdot p_{B_c^+}\big)A_0^{B_c^+\to J/\psi}(m_\pi^2)a_1(J/\psi\pi)\Big]^2} \times\Big\{m_{J/\psi}\big(\epsilon_{J/\psi}\cdot p_{B_c^+}\big)\Big[f_{D_s^+}A_0^{B_c^+\to J/\psi}\Big(m^2_{D_s^+}\Big)a_1(J/\psi D_s^+) + f_{J/\psi}F_1^{B_c^+\to D_s^+}(m^2_{J/\psi})a_2\big(J/\psi D_s^+\big)\Big] + \frac{1}{2}f_{B_c^+}f_{J/\psi}f_{D_s^+}b_2\Big\}^2. \quad (9)$$

The theoretical predictions depend on many input parameters such as Wilson coefficients, the CKM matrix elements, masses, lifetimes, decay constants, form factors, and so on. We present all the relevant input parameters as follows:

Wilson coefficients, the Wilson coefficients c_1 and c_2 in the effective weak Hamiltonian have been reliably evaluated to the next-to-leading logarithmic order. To proceed, we use the following numerical values at $\mu = m_b$ scale, which have been obtained in the NDR scheme:[16] $c_1 = 1.081$ and $c_2 = -0.190$.

The CKM matrix elements, the Cabibbo–Kobayashi–Maskawa (CKM) matrix is a 3×3 unitary matrix, the elements of this matrix can be parametrized by three mixing angles A, λ, ρ and a CP-violating phase η:[19] $V_{ud} = 1 - \lambda^2/2$, $V_{us} = \lambda$, $V_{ub} = A\lambda^3(\rho - i\eta)$, $V_{cd} = -\lambda$, $V_{cs} = 1 - \lambda^2/2$, $V_{cb} = A\lambda^2$, $V_{td} = A\lambda^3(1 - \rho - i\eta)$, $V_{ts} = -A\lambda^2$ and $V_{tb} = 1$. The results for the Wolfenstein parameters are $\lambda = 0.22537 \pm 0.00061$, $A = 0.814^{+0.023}_{-0.024}$, $\bar{\rho} = 0.117 \pm 0.021$ and $\bar{\eta} = 0.353 \pm 0.013$. We use the values of the Wolfenstein parameters and obtain $V_{ud} = 0.97427 \pm 0.00014$, $V_{us} = 0.22536 \pm 0.00061$, $V_{ub} = 0.00355 \pm 0.00015$, $V_{cd} = 0.22522 \pm 0.00061$, $V_{cs} = 0.97343 \pm 0.00015$, $V_{cb} = 0.0414 \pm 0.0012$, $V_{td} = 0.00886^{+0.00033}_{-0.00032}$, $V_{ts} = 0.0405^{+0.0011}_{-0.0012}$ and $V_{tb} = 0.99914 \pm 0.00005$.

Masses and decay constants (in units of MeV), the meson masses and decay constants needed in our calculations are taken as:[19] $m_{\psi(2S)} = 3686.109 \pm 0.012$, $m_{J/\psi} = 3096.916 \pm 0.011$, $m_{D_s^+} = 1968.30 \pm 0.10$, $m_{K^+} = 493.677 \pm 0.016$, $m_{\pi^+} = 139.57018 \pm 0.00035$, $m_b = 4190^{+180}_{-60}$, $m_c = 1290^{+50}_{-110}$, $m_s = 100^{+30}_{-20}$, $f_{B_c^+} = 489 \pm 4$,[20] $f_{J/\psi} = 418 \pm 9$, $f^{\perp}_{J/\psi} = (2m_c/m_{J/\psi})f_{J/\psi} = 348 \pm 7.5$,[21] $f_{D_s^+} = 257.5 \pm 4.6$, $f_K = 156.2 \pm 0.2$, $f_\pi = 130.41 \pm 0.03$ and $\Lambda_{\rm QCD} = 225$.

Form factors, for the parameters σ_1, σ_2 and $f(0)$ used in transition weak form factors, we take:[22] $f(0) = 0.59$, $\sigma_1 = 0.047$, $\sigma_2 = 0.0017$ for $A_0^{B_c^+ \to J/\psi}(q'^2)$; $f(0) = 0.46$, $\sigma_1 = 0.038$, $\sigma_2 = 0.0015$ for $A_1^{B_c^+ \to J/\psi}(q'^2)$ and $f(0) = 0.64$, $\sigma_1 = 0.064$, $\sigma_2 = 0.0041$ for $A_2^{B_c^+ \to J/\psi}(q'^2)$; for the form factor involving the $A_0^{B_c^+ \to J/\psi}(m_\pi^2)$ transition, we use $A_0^{B_c^+ \to J/\psi}(m_\pi^2) = 0.59$ and also $F_1^{B_c^+ \to D_s^+}(0) = 0.15 \pm 0.01$.[18]

In the literature, there already exist some studies on B_c^+ decays to the $J/\psi\pi^+$ and $J/\psi K^+$ final mesons. In the following, we are interested in declaring the values obtained for the branching fraction of the $B_c^+ \to J/\psi\pi^+$ and $B_c^+ \to J/\psi K^+$ decays, so we can compare our results with that in the literatures. For these decay modes, we obtain $\mathcal{B}(B_c^+ \to J/\psi\pi^+) = 2.44 \times 10^{-3}$ which is in good agreement with the upper limit of the span the range $[0.34, 2.9] \times 10^{-3}$,[23] and is comparable with the previous works, 1.1×10^{-3},[24] 1.3×10^{-3},[25] and $1.7, 1.8 \times 10^{-3}$;[26,27] and $\mathcal{B}(B_c^+ \to J/\psi K^+) = 1.81 \times 10^{-4}$ is comparable with these works, 1.3×10^{-4},[27] and 1.4×10^{-4}.[26]

The relative branching fractions of the B_c^+ decays are calculated to be

$$\begin{aligned} \frac{\mathcal{B}(B_c^+ \to \psi(2S)\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} &= 0.327 \pm 0.028\,, \\ \frac{\mathcal{B}(B_c^+ \to J/\psi K^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} &= 0.074 \pm 0.0057\,, \\ \frac{\mathcal{B}(B_c^+ \to J/\psi D_s^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} &= 3.257 \pm 0.293\,. \end{aligned} \tag{10}$$

For the first ratio of Eq. (10), the value of 0.260 has been estimated in the literature.[28] Also, in the literatures[28] and[29] numerical results for the ratio of the $\mathcal{B}(B_c^+ \to J/\psi K^+)$ to that $\mathcal{B}(B_c^+ \to J/\psi\pi^+)$ to be 0.075 and 0.078, respectively, are comparable to our second achievement, summarized in (10). It should be noted that our calculations depend on many input parameters, among the experimental data with error bars and the theoretical values including large uncertainties. We consider the main uncertainties arising from the uncertainties of the input parameters of the CKM matrix elements, the form factors, the decay constants and the meson masses.

4. Conclusion

In this study, we have presented a comprehensive calculation of the $B_c^+ \to \psi(2S)\pi^+$, $B_c^+ \to J/\psi K^+$, $B_c^+ \to J/\psi D_s^+$ and $B_c^+ \to J/\psi\pi^+$ decays. In fact, we have been interested in examining the branching ratio of the first three decays relative to the $B_c^+ \to J/\psi\pi^+$ decay. In the calculation of the decay amplitudes, we have considered the "factorizable" contributions and "nonfactorizable" corrections, the "factorizable" contributions are calculated via three terms: (i) the current-induced process with a meson emission, (ii) the transition process and (iii) the annihilation process. Under the factorization approach, the amplitude of the $B_c^+ \to J/\psi\pi^+$ decay has been calculated by using the color-allowed external W-emission a_1 tree diagram, all the "nonfactorizable" effects (coming from the vertex-correction and hard spectator-scattering diagrams) are encoded in the coefficients a_1, which are process-dependent and can be obtained perturbatively. The ratios between the $\mathcal{B}(B_c^+ \to \psi(2S)\pi^+)$, $\mathcal{B}(B_c^+ \to J/\psi K^+)$, $\mathcal{B}(B_c^+ \to J/\psi D_s^+)$ and $\mathcal{B}(B_c^+ \to J/\psi\pi^+)$ are 0.327 ± 0.028, 0.074 ± 0.0057 and 3.257 ± 0.293, respectively, compatible with the experimental data.

References

1. CDF Collab. (F. Abe *et al.*), *Phys. Rev. Lett.* **81**, 2432 (1998).
2. CDF Collab. (T. Aaltonen *et al.*), *Phys. Rev. Lett.* **100**, 182002 (2008).
3. D0 Collab. (V. Abazov *et al.*), *Phys. Rev. Lett.* **101**, 012001 (2008).
4. LHCb Collab. (R. Aaij *et al.*), *Phys. Rev. D* **87**, 071103 (2013).
5. LHCb Collab. (R. Aaij *et al.*), *Phys. Rev. Lett.* **111**, 181801 (2013).

6. I. P. Gouz, V. V. Kiselev, A. K. Likhoded, V. I. Romanovsky and O. P. Yushchenko, *Phys. Atom. Nucl.* **67**, 1559 (2004).
7. LHCb Collab. (R. Aaij *et al.*), *Phys. Rev. D* **94**, 091102 (2016).
8. B. Mohammadi, *Nucl. Phys. A* **969**, 196 (2018).
9. LHCb Collab. (R. Aaij *et al.*), *Phys. Rev. D* **92**, 072007 (2015).
10. LHCb Collab. (R. Aaij *et al.*), *J. High Energy Phys.* **09**, 075 (2013).
11. LHCb Collab. (R. Aaij *et al.*), *J. High Energy Phys.* **1609**, 153 (2016).
12. LHCb Collab. (R. Aaij *et al.*), *Phys. Rev. D* **87**, 112012 (2013).
13. B. Mohammadi, *AIP Conf. Proc.* **1819**, 060011 (2017).
14. M. Beneke and S. Jager, *Nucl. Phys. B* **751**, 160 (2006).
15. M. Beneke and M. Neubert, *Nucl. Phys. B* **675**, 333 (2003).
16. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, *Nucl. Phys. B* **606**, 245 (2001).
17. Z. G. Wang, *Commun. Theor. Phys.* **61**, 81 (2014).
18. R. Dhir and R. C. Verma, *Phys. Rev. D* **79**, 034004 (2009).
19. Particle Data Group (K. A. Olive), *Chin. Phys. C* **38**, 090001 (2014).
20. TWQCD Collab. (T. W. Chiu and T. H. Hsieh), *PoS* **LAT 2006**, 180 (2007).
21. H. Y. Cheng and K. C. Yang, *Phys. Rev. D* **63**, 074011 (2001).
22. W. F. Wang, Y. Y. Fan and Z. J. Xiao, *Chin. Phys. C* **37**, 093102 (2013).
23. LHCb Collab. (R. Aaij *et al.*), *Phys. Rev. D* **94**, 091102 (2016).
24. A. Abd El-Hady, J. H. Munoz and J. P. Vary, *Phys. Rev. D* **62**, 014019 (2000).
25. P. Colangelo and F. De Fazio, *Phys. Rev. D* **61**, 034012 (2000).
26. C. H. Chang and Y. Q. Chen, *Phys. Rev. D* **49**, 3399 (1994).
27. M. A. Ivanov, J. G. Korner and P. Santorelli, *Phys. Rev. D* **73**, 054024 (2006).
28. C. F. Qiao, P. Sun, D. Yang and R. L. Zhu, *Phys. Rev. D* **89**, 034008 (2014).
29. D. Ebert, R. N. Faustov and V. O. Galkin, *Phys. Rev. D* **68**, 094020 (2003).