Published for SISSA by Springer

Received: September 6, 2017
Revised: November 17, 2017
Accepted: November 26, 2017
Published: December 4, 2017

Study of B_c^+ decays to the $K^+K^-\pi^+$ final state by using B_s^0, χ_{c0} and D^0 resonances and weak annihilation nonresonant topologys

Behnam Mohammadi

Department of Physics, University of Urmia, Nazlu Campus, Urmia, Iran

E-mail: be.mohammadi@urmia.ac.ir

Abstract: In this research the weak decay of B_c^+ decays to the $K^+K^-\pi^+$ final state, which is being observed by LHCb collaboration for the first time, is calculated in the quasi-two-body decays which takes the B_s^0, χ_{c0} and D^0 resonances and weak annihilation nonresonant contributions into account. In this process, the B_c^+ meson decays first into $B_s^0\pi^+$, $\chi_{c0}\pi^+$ and $D^0\pi^+$ intermediate states, and then the B_s^0, χ_{c0} and D^0 resonances decay into K^+K^- components, which undergo final state interaction. The mode of the $B_c^+ \to D^0(\to K^-\pi^+)K^+$ is also associated with the calculation, in this mode the intermediate resonance D^0 decays to the $K^-\pi^+$ final mesons. The resonances B_s^0, χ_{c0} and D^0 effects in the $B_c^+ \to B_s^0(\to K^+K^-)\pi^+$, $B_c^+ \to \chi_{c0}(\to K^+K^-)\pi^+$ and $B_c^+ \to D^0(\to K^+K^-)\pi^+, D^0(\to K^-\pi^+)K^+$ decays are described in terms of the quasi-two-body modes. There is a weak annihilation nonresonant contribution in which B_c^+ decays to the $K^+K^-\pi^+$ directly, so the point-like 3-body matrix element $\langle K^+K^-\pi^+|u\bar{d}|0\rangle$ is also considered. The decay mode of the $B_c^+ \to \bar{K}^{*0}(892)K^+$ is contributed to the annihilation contribution. The branching ratios of quasi-two-body decays expand in the range of $(2.12 \pm 0.61) \times 10^{-6}$ to $(7.56 \pm 1.71) \times 10^{-6}$.

Keywords: Beyond Standard Model, Heavy Quark Physics

ArXiv ePrint: 1709.00550

https://doi.org/10.1007/JHEP12(2017)019

Contents

1 Introduction

B_c meson is one of the most interesting mesons that can be studied at the Tevatron, the discovery of the B_c was reported by the CDF collaboration in the $B_c \to J/\psi l^{\pm} \bar{\nu}_l$ process at Fermilab [1]. After that, the decay mode $B_c^{\pm} \to J/\psi \pi^{\pm}$ has been observed by CDF and D0 collaboration significance of more than 8σ and 5σ respectively [2, 3]. This decay mode addition $B_c^{\pm} \to J/\psi D_s^{\pm}$ decay have also been observed by the LHCb collaboration at the LHC center-of-mass energy 7 TeV of proton-proton collisions [4, 5]. Studies of B_c properties are important, because it is made of two different heavy quarks, bottom-charm antiquark-quark pair. Each of the quarks can participate in a weak interaction in which other quark participates as a spectator. For this, B_c is also the only meson in which decays of both heavy quarks compete with each other, therefore a wide range of decay channels are possible. However, a significant number of these channels has not been observed yet [6]. Unlike B^0, B^+ and B_s^0 mesons, more than 70% of the B_c^+ width is due to c-quark decays, in which $c \to s$ transition has been observed with $B_c^+ \to B_s^0 \pi^+$ decays [7]. Around 20% of its width is due to the b-quark decays [8]. In charmless final states, the $\bar{b}c \to W^+ \to \bar{q}q$ annihilation amplitudes account for only 10% of the B_c^+ width [9].

The mass of the B_c meson has been predicted using a variety of theoretical techniques. Nonrelativistic potential models have been used to predict a mass of the B_c in the range of 6247–6286 MeV/c2 [10–12], by using a perturbative QCD calculation slightly higher value is found [13] and recent a B_c mass prediction of $6304 \pm 12^{+18}_{-0}$ MeV/c2 have been provided applying lattice QCD calculations [14], in which are heavier than that of other B mesons, this suggests an expected lifetime much shorter than those as $\tau_{B_c^+} = 513.4 \pm 11.0 \pm 5.7$ fs [15, 16].

The decay of B_c^+ meson to three light charged hadrons like K^+, K^- and π^+, which is observed by LHCb collaboration for the first time [9], provides a good way to study standard model for which has a large available phase space. In this work the decay mode of $B_c^+ \to K^+K^-\pi^+$ is being studied which includes other processes such as $B_c^+ \to B_s^0(\to K^+K^-)\pi^+$ decay mediated by $c \to s$ transition, charmonium mode $B_c^+ \to \chi_{c0}(\to K^+K^-)\pi^+$ mediated by the $\bar{b} \to \bar{c}$ transition, $B_c^+ \to D^0(\to K^+K^-)\pi^+$ mediated by the $\bar{b} \to \bar{u}$ and $\bar{b} \to \bar{d}$ transitions and finally the decay of $B_c^+ \to D^0(\to K^-\pi^+)K^+$ mediated by the $\bar{b} \to \bar{u}$ and $\bar{b} \to \bar{s}$ transitions. In the standard model, there is another process that can be mediated via $c\bar{b} \to W^+ \to u\bar{d}$ annihilation topology, in this mode the B_c^+ decays with no charm and beauty particles in the final or intermediate states. In the B_c^+ region $6.0 < m(K^+K^-\pi^+) < 6.5\,\mathrm{GeV}/c^2$, the signals were fitted by authors in ref. [9] separately for regions of the phase space corresponding to the different expected contributions: (a) the annihilation region, $m(K^-\pi^+) < 1.834\,\mathrm{GeV}/c^2$, they have claimed the $\bar{K}^{*0}(892)$ meson can be found in this region. The contribution of the mode $B_c^+ \to \bar{K}^{*0}(892)(\to K^-\pi^+)K^+$ and direct annihilation are obtained separately which the estimation of $\bar{K}^{*0}(892) \to K^-\pi^+$ is lower than the direct annihilation calculation, (b) the $D^0 \to K^-\pi^+$ region, $1.834 < m(K^-\pi^+) < 1.894\,\mathrm{GeV}/c^2$, (c) the $B_s^0 \to K^+K^-$ region, $5.3 < m(K^+K^-) < 5.4\,\mathrm{GeV}/c^2$, (d) the $\chi_{c0} \to K^+K^-$ region, $3.38 < m(K^+K^-) < 3.46\,\mathrm{GeV}/c^2$. A concentration of events was observed by [9] around $m^2(K^+K^-) \sim 11\mathrm{GeV}^2/\mathrm{c}^4$, a one-dimensional projection of $m(K^+K^-)$ shows clustering near $3.41\,\mathrm{GeV}/c^2$, close to the mass of the charmonium state χ_{c0} where it has the highest branching fraction into the K^+K^- final state [17]. The accumulation of events was also observed near $m^2(K^+K^-) \sim 29\mathrm{GeV}^2/\mathrm{c}^4$ close to the mass of the B_s^0 meson. In this research the study of the $B_c^+ \to K^+K^-\pi^+$ via quasi-two-body decay was considered, this decay mode was observed in the $B_s^0 \to K^+K^-$, $\chi_{c0} \to K^+K^-$, $D^0 \to K^+K^-$ and $D^0 \to K^-\pi^+$ channels. It is known that in the narrow width approximation, in the models we use to obtain the amplitudes of the decays, the 3-body decay rate obeys the factorization relation [18]

$$\mathcal{B}(B_c^+ \to RM \to M_1M_2M) = \mathcal{B}(B_c^+ \to RM) \times \mathcal{B}(R \to M_1M_2), \tag{1.1}$$

with R being B_s^0, χ_{c0} and D^0 intermediate resonance mesons and M, M_1 and M_2 are K^+, K^- and π^+ final state mesons. The intermediate resonance effects are described in terms of the Breit-Wigner formalism. The Breit-Wigner resonant term associated to quasi two body $R + M$ state which seems to play an important role as indicated by experiments. We have to calculate the branching ratios of the $\mathcal{B}(B_c^+ \to RM)$ and $\mathcal{B}(R \to M_1M_2)$ by using the Feynman quark diagrams.

2 Decay amplitudes and branching fractions

In the standard factorization scheme, the decay amplitude is obtained by applying weak Hamiltonian which is given below:

$$A(P \to P_1P_2) = \langle P_1|J^\mu|0\rangle\langle P_2|J_\mu^\dagger|P\rangle + \langle P_2|J^\mu|0\rangle\langle P_1|J_\mu^\dagger|P\rangle \tag{2.1}$$

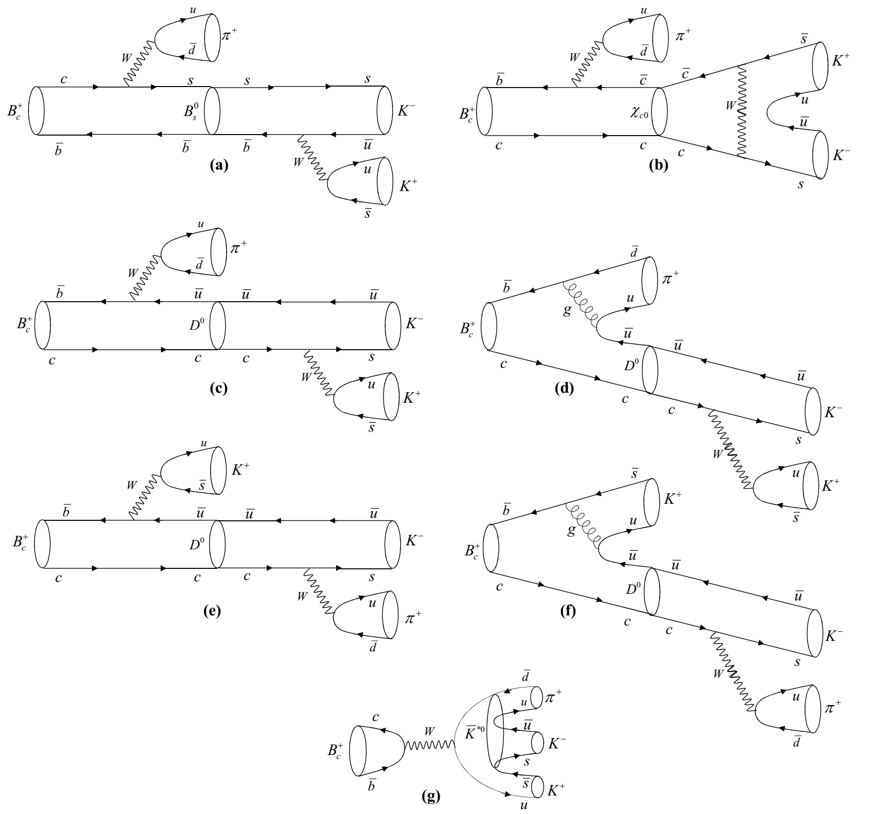

Figure 1. Feynman diagrams for $B_c^+ \to K^+K^-\pi^+$ decays using the (a) $B_c^+ \to B_s^0(\to K^+K^-)\pi^+$ channel mediated by $c \to s$ transition; (b) $B_c^+ \to \chi_{c0}(\to K^+K^-)\pi^+$ channel mediated by $\bar{b} \to \bar{c}$ transition; (c) and (d) $B_c^+ \to D^0(\to K^+K^-)\pi^+$ channel mediated by $\bar{b} \to \bar{u}$ and $\bar{b} \to \bar{d}$ transitions; (e) and (f) $B_c^+ \to D^0(\to K^-\pi^+)K^+$ channel mediated by $\bar{b} \to \bar{u}$ and $\bar{b} \to \bar{s}$ transitions; (g) annihilation process.

where the weak current J_μ is given by: $(\bar{u}\ \bar{c}\ \bar{t})\gamma_\mu(1-\gamma_5)\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$, here d', s' and b' are mixture of the d, s and b quarks, as given by the CKM matrix. Current matrix elements are defined as: $\langle P_{1,2}(p_{1,2})|J_\mu|P(p)\rangle = (p+p_{1,2}-q(m_P^2-m_{p_{1,2}}^2)/q^2)_\mu F_1(q^2)+q_\mu(m_P^2-m_{p_{1,2}}^2)/q^2 F_0(q^2)$ with $q_\mu = p_\mu - p_{(1,2)\mu}$ and $\langle P_{2,1}(p_{2,1})|J^\mu|0\rangle = if_{p_{2,1}}p_{2,1}^\mu$. It has been pointed out in the BSW2 model [19, 20] that consistency with the heavy quark symmetry requires certain form factors such as F_0 and F_1 to have dipole q^2 dependence i.e. $F_{0,1}^{P\to P_{1,2}}(q^2) = F_{0,1}^{P\to P_{1,2}}(0)/(1-q^2/m_V^2)^2$, as an example for the $c \to b$ transition m_V is $m_{B_c^*}$. In the following, we calculate the amplitudes of the all decay modes. Different with $B_{u,d,s}$ mesons, the B_c^+ system consists of two heavy quarks $\bar{b}$ and c, which can decay individually. Here we will consider $\bar{b}$ decays while c acts as a spectator except $B_c^+ \to B_s^0\pi^+$ decay in which we have $c \to s$ transition. According to figure 1, at the quark level corresponding effective Hamiltonian is given by: $H_{\rm eff} = (G_F/\sqrt{2})\sum_{p=u,c}\lambda_p(c_1Q_1^p+c_2Q_2^p+\sum_{i=3,\dots,10}c_iQ_i+c_{7\gamma}q_{7\gamma}+c_{8g}Q_{8g})$ where λ_p is the CKM matrix elements, c_i are the Wilson coefficients evaluated at the renormalization scale μ, $Q^p_{1,2}$ are the left-handed current-current operators arising from W-boson exchange, $Q_{3,\ldots,6}$ and $Q_{7,\ldots,10}$ are QCD and electroweak penguin operators, and $Q_{7\gamma}$ and Q_{8g} are the electromagnetic and chromomagnetic dipole operators. Because in the tree level diagrams (a_1 coefficient) both $\bar{b} \to \bar{u}$ and $\bar{b} \to \bar{c}$ transitions are available, so we have considered to current-current operators Q^u_1, Q^u_2, Q^c_1 and Q^c_2 as: $Q^u_1 = (\bar{b}_\alpha u_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A}$, $Q^u_2 = (\bar{b}_\alpha u_\beta)_{V-A}(\bar{d}_\beta u_\alpha)_{V-A}$, $Q^c_1 = (\bar{b}_\alpha c_\alpha)_{V-A}(\bar{d}_\beta u_\beta)_{V-A}$ and $Q^c_2 = (\bar{b}_\alpha u_\beta)_{V-A}(\bar{d}_\beta c_\alpha)_{V-A}$. The operators arise from the QCD-penguin diagrams (both $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ transitions) which contribute in the a_4 coefficient we have considered: $Q_3 = (\bar{b}_\alpha d_\alpha(s_\alpha))_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$, $Q_4 = (\bar{b}_\alpha d_\beta(s_\beta))_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$. Here α and β are the SU(3) color indices and the subscript $V-A$ represent the chiral projection $1-\gamma_5$.

2.1 $B^+_c \to B^0_s\pi^+$ decay

In the following, we calculate the amplitude of the $B^+_c \to B^0_s\pi^+$ decay mediated by $c \to s$ transition. The amplitude of the $B^+_c \to B^0_s\pi^+$ decay by using the color-allowed external W-emission tree diagram become: $(G_F/\sqrt{2})V_{cs}V^*_{ud}\langle B^0_s|(c\bar{s})_{V-A}|B^+_c\rangle\langle\pi^+|(u\bar{d})_{V-A}|0\rangle_F$, where $_F$ denote the factorized hadronic matrix element, which has the same definition as that in the "nonfactorizable" approach. The factorized matrix element of the $\langle B^0_s(p_1)|(c\bar{s})_{V-A}|B^+_c(p)\rangle$ can be defined as

$$\left(p_\mu + p_{1\mu} - \frac{m^2_{B^+_c} - m^2_{B^0_s}}{q^2}q_\mu\right) F_1^{B^+_c\to B^0_s}(m^2_\pi) + \frac{m^2_{B^+_c} - m^2_{B^0_s}}{q^2}q_\mu F_0^{B^+_c\to B^0_s}(m^2_\pi) \tag{2.2}$$

and the decay constant of the $\langle\pi^+(p_2)|(u\bar{d})_{V-A}|0\rangle$ become: $if_\pi p^\mu_2$, where $q_\mu = p_{2\mu} = p_\mu - p_{1\mu}$, after multiplying the form factor in the decay constant only the $F_0^{B^+_c\to B^0_s}(m^2_\pi)$ component remains in the amplitude which is calculated from the $F_0^{B^+_c\to B^0_s}(m^2_\pi) = F_0^{B^+_c\to B^0_s}(0)/(1 - m^2_\pi/m^2_{\rm fit})^2$ [21]. Since the decay of $B^+_c \to B^0_s\pi^+$ mediated by $c \to s$ transition so the $m_{\rm fit}$ becomes: $m_{D^{*+}_s}$. The expressions of decay amplitude for $B^+_c \to B^0_s\pi^+$ within the factorization framework can be written as

$$A(B^+_c \to B^0_s\pi^+) = i\frac{G_F}{\sqrt{2}}a_1 V_{cs}V^*_{ud}f_\pi(m^2_{B^+_c} - m^2_{B^0_s})F_0^{B^+_c\to B^0_s}(m^2_\pi), \tag{2.3}$$

where $a_1 = c_1 + c_2/3$. The branching fraction of $B^+_c \to B^0_s\pi^+$ in B^+_c meson rest frame can be written as

$$\mathcal{B}(B^+_c \to B^0_s\pi^+) = \frac{\tau_{B^+_c}}{8\pi}\frac{|\vec{p}|}{m^2_{B^+_c}}|A(B^+_c \to B^0_s\pi^+)|^2, \tag{2.4}$$

in which where $\tau_{B^+_c}$ is the lifetime of the B^+_c meson and $|\vec{p}|$ is the absolute value of the 3-momentum of the B^0_s or π^+ mesons that can be calculated via:

$$\sqrt{(m^2_{B^+_c} + m^2_{B^0_s} - m^2_\pi)^2 - 4m^2_{B^+_c}m^2_{B^0_s}}/(2m_{B^+_c})\,.$$

2.2 $B^+_c \to \chi_{c0}\pi^+$ decay

The same calculation, similar to the pervious calculation, can also be applied to the $B^+_c \to \chi_{c0}\pi^+$ mode, with the difference that the state of decay mediated by $\bar{b} \to \bar{c}$ transition so the factorized matrix element of the $\langle\chi_{c0}(p_1)|(c\bar{b})_{V-A}|B^+_c(p)\rangle$ related to the form

JHEP12(2017)019

factor of $F_0^{B_c^+\to\chi_{c0}}(m_\pi^2)$ so that calculates with the $F_0^{B_c^+\to\chi_{c0}}(0)/(1-m_\pi^2/m_{B_c^{*+}}^2)^2$, here $m_{\rm fit}$ is $m_{B_c^{*+}}$ for $\bar{b}\to\bar{c}$ transition. The amplitude and branching fraction of the $B_c^+\to\chi_{c0}\pi^+$ is the same with the $B_c^+\to B_s^0\pi^+$ one which are given by

$$A(B_c^+\to\chi_{c0}\pi^+) = i\frac{G_F}{\sqrt{2}}a_1V_{cb}V_{ud}^*f_\pi(m_{B_c^+}^2-m_{\chi_{c0}}^2)F_0^{B_c^+\to\chi_{c0}}(m_\pi^2), \tag{2.5}$$

and

$$\mathcal{B}(B_c^+\to\chi_{c0}\pi^+) = \frac{\tau_{B_c^+}}{8\pi}\frac{|\vec{p}|}{m_{B_c^+}^2}|A(B_c^+\to\chi_{c0}\pi^+)|^2, \tag{2.6}$$

to obtain the absolute value of the 3-momentum of the χ_{c0} or π^+ mesons, the χ_{c0} meson mass should be replaced with the B_s^0 meson mass in the pervious calculation. The branching ratios for other decays $B_c^+\to D^0\pi^+$, $B_c^+\to D^0K^+$, $B_s^0\to K^+K^-$, $D^0\to K^+K^-$ and $D^0\to K^-\pi^+$ have the similar expressions as eqs. (2.4) and (2.6)

2.3 $B_c^+\to D^0\pi^+$ and $B_c^+\to D^0K^+$ decays

These decays, in addition $\bar{b}\to\bar{u}$ tree level a_1 coefficient, involve the $\bar{b}\to\bar{d}$ and $\bar{b}\to\bar{s}$ penguin amplitudes (a_4 coefficient) with the QCD penguins participating. The same amplitudes of the $B_c^+\to D^0\pi^+$ and $B_c^+\to D^0K^+$ decays are given by

$$\begin{aligned}A\big(B_c^+\to D^0\pi^+(K^+)\big) = i\frac{G_F}{\sqrt{2}}&\big[a_1V_{ub}V_{ud}^*(V_{us}^*)-a_4V_{tb}V_{td}^*(V_{ts}^*)\big]\\ &f_\pi\big(f_K\big)(m_{B_c^+}^2-m_{D^0}^2)F_0^{B_c^+\to D^0}(m_\pi^2(m_K^2)),\end{aligned} \tag{2.7}$$

where $a_4=c_4+c_3/3$ and for $\bar{b}\to\bar{u}$ transition we use $F_0^{B_c^+\to D^0}(m_\pi^2(m_K^2)) = F_0^{B_c^+\to D^0}(0)/(1-m_\pi^2(m_K^2)/m_{B^{*+}}^2)^2$.

2.4 $B_s^0\to K^+K^-$, $D^0\to K^+K^-$ and $D^0\to K^-\pi^+$ decays

In the factorization approach the amplitudes of the $B_s^0\to K^+K^-$, $D^0\to K^+K^-$ and $D^0\to K^-\pi^+$ decays have the following form

$$A(B_s^0\to K^+K^-) = i\frac{G_F}{\sqrt{2}}a_1V_{ub}V_{us}^*f_{K^+}(m_{B_s^0}^2-m_{K^-}^2)F_0^{B_s^0\to K^-}(m_{K^+}^2), \tag{2.8}$$

$$A(D^0\to K^+K^-) = i\frac{G_F}{\sqrt{2}}a_1V_{cs}V_{us}^*f_{K^+}(m_{D^0}^2-m_{K^-}^2)F_0^{D^0\to K^-}(m_{K^+}^2), \tag{2.9}$$

$$A(D^0\to K^-\pi^+) = i\frac{G_F}{\sqrt{2}}a_1V_{cs}V_{ud}^*f_{\pi^+}(m_{D^0}^2-m_{K^-}^2)F_0^{D^0\to K^-}(m_{\pi^+}^2). \tag{2.10}$$

For $\bar{b}\to\bar{u}$ transition of $B_s^0\to K^+K^-$ decay we use $F_0^{B_s^0\to K^-}(m_{K^+}^2) = F_0^{B_s^0\to K^-}(0)/(1-m_{K^+}^2/m_{B^{*+}}^2)^2$ and for $c\to s$ transition of $D^0\to K^-K^+(\pi^+)$ decays we apply $F_0^{D^0\to K^-}(m_{K^+}^2(m_{\pi^+}^2)) = F_0^{D^0\to K^-}(0)/(1-m_{K^+}^2(m_{\pi^+}^2)/m_{D_s^{*+}}^2)^2$.

JHEP12(2017)019

2.5 Annihilation topology

In this subsection we offer two ways for annihilation topology, the first is two body pure annihilation topology for $\chi_{c0} \to K^+K^-$ decay, this decay mode proceeds only through the W-annihilation diagram and the second is nonresonant three body $B_c^+ \to K^+K^-\pi^+$ annihilation contribution. The contribution of the mode $B_c^+ \to \bar{K}^{*0}(892)K^+$ in the annihilation processes could be also prominent.

2.5.1 Pure annihilation $\chi_{c0} \to K^+K^-$ decay

The $\chi_{c0} \to K^+K^-$ decay is a pure annihilation decay channel, this process only occurs via annihilation between c and s quarks. In the factorization method, Feynman diagram for the $\chi_{c0} \to K^+K^-$ decay is shown in figure 1. When all the basic building blocks equations are solved, for the case that both mesons are pseudoscalar, it is found that weak annihilation Kernels exhibit endpoint divergence. Divergence terms are determined by $\int_0^1 dx/\bar{x}$ and $\int_0^1 dy/y$. For the liberation of the divergence, a small ϵ of $\Lambda_{\rm QCD}/m_{\chi_{c0}}$ order was added to the denominator. So the answer to the integral becomes $\ln(1+\epsilon)/\epsilon$ form, which is shown with X_A. Specifically, we treat $X_A = (1+\rho_A e^{i\varphi_A})\ln(m_{\chi_{c0}}/\Lambda_{\rm QCD})$ as a arbitrary parameter obtained by using $\rho_A = 0.5$ and a strong phase $\varphi_A = -55^\circ$ [22]. The factorization amplitude when both mesons are pseudoscalar is given by

$$A(\chi_{c0} \to K^+K^-) = i\frac{G_F}{\sqrt{2}} b_1 V_{cs} V_{cs}^* f_{\chi_{c0}} f_{K^+} f_{K^-}, \tag{2.11}$$

where b_1 is the building block of the non-singlet annihilation coefficient which is given by: $(8/27)\pi\alpha_s c_1\big[9(X_A - 4 + \pi^2/3) + r_\chi^{K^+} r_\chi^{K^-} X_A^2\big]$. The light-cone expansion implies that only leading-twist distribution amplitudes are needed in the heavy-quark limit. There exist however a number of subleading quark-antiquark distribution amplitudes of twist 3, which have large normalization factors for pseudoscalar mesons, e.g. r_χ^K for the kaon we use $2m_K^2/\big((m_c - m_u)(m_u + m_s)\big)$

2.5.2 Effects of $B_c^+ \to \bar{K}^{*0}(892)K^+$ decay to the annihilation processes

According to the panel (g) of the figure 1, the decay mode of the $B_c^+ \to \bar{K}^{*0}(892)K^+$ could be contributed to the annihilation processes. The amplitude of the $B_c^+ \to \bar{K}^{*0}(892)K^+$ annihilation decay reads

$$A(B_c^+ \to \bar{K}^{*0}K^+) = i\frac{G_F}{\sqrt{2}} b_2 V_{cb} V_{ud}^* f_{B_c^+} f_{\bar{K}^{*0}} f_{K^+}, \tag{2.12}$$

when one of the final state mesons is vector and another is pseudoscalar, the building block of the non-singlet annihilation coefficient b_2 becomes: $(8/9)\pi\alpha_s c_2\big[3(X_A - 4 + \pi^2/3) + r_\chi^{\bar{K}^{*0}} r_\chi^{K^+}(X_A^2 - 2X_A)\big]$, the ratio of the $r_\chi^{\bar{K}^{*0}}$ is calculated via: $(2m_{\bar{K}^{*0}}/m_b)(f_{\bar{K}^{*0}}^\perp/f_{\bar{K}^{*0}})$.

2.5.3 Nonresonant annihilation contribution for $B_c^+ \to K^+K^-\pi^+$ decay

As for the three-body nonresonant annihilation matrix element $\langle K^+(p_1)K^-(p_2)\pi^+(p_3)|(u\bar{d})_{V-A}|0\rangle$, we can show that it vanishes in the chiral limit owing to the helicity suppression. To prove this claim, we first assume that the kaon and pion mesons are soft.Tthe point-like 3-body matrix element is chiral-realization dependent, this realization dependence should be compensated by the pole contribution, so the three-body nonresonant annihilation matrix element becomes: $(2i/f_\pi)\big(p_{2\mu} - p_\mu(p \cdot p_2)/(p^2 - m_K^2)\big)$. It is worth stressing again that the above matrix element is valid only for low-momentum pseudoscalars. It is easily seen that in the chiral limit the $\langle K^+(p_1)K^-(p_2)\pi^+(p_3)|(u\bar{d})_{V-A}|0\rangle\langle 0|(c\bar{b})_{V-A}|B_c^+(p)\rangle$ become $\big((m_K^2 - m_\pi^2)/(m_s - m_d)\big)\big((f_{B_c^+}m_{B_c^+}^2)/(f_{\pi^+}m_b)\big)\big(1 - (2p_1 \cdot p_3)/(m_{B_c^+}^2 - m_K^2)\big)F^{KK\pi}(m_{B_c^+}^2)$, where the form factor $F^{KK\pi}(m_{B_c^+}^2)$ is needed to accommodate the fact that the final-state pseudoscalars are energetic rather than soft that is assumed to be $1/[(1 - m_{B_c^+}^2/\Lambda_\chi^2)]$, with $\Lambda_\chi = 830\,\mathrm{MeV}$ is the chiral-symmetry breaking scale. The direct three body weak annihilation amplitude reads

$$A(B_c^+ \to K^+K^-\pi^+)_{\mathrm{ann}} = i\frac{G_F}{\sqrt{2}}a_1 V_{cb}V_{ud}^* \frac{f_{B_c^+}}{f_{\pi^+}}\frac{m_K^2 - m_\pi^2}{m_s - m_d}\left(1 - \frac{2p_1 \cdot p_3}{m_{B_c^+}^2 - m_K^2}\right)F^{KK\pi}(m_{B_c^+}^2). \tag{2.13}$$

To obtain the $p_1 \cdot p_3$, we consider the decays of B_c^+ meson (with 4-momentum of p and m mass) into three K^+, K^- and π^+ particles. Denote their masses m_1, m_2 and m_3 and 4-momenta by p_1, p_2 and p_3, respectively. Energy-momentum conservation is expressed by $p = p_1 + p_2 + p_3$. Define the following invariants $s_{12} = (p_1 + p_2)^2 = (p - p_3)^2$, $s_{13} = (p_1 + p_3)^2 = (p - p_2)^2$, $s_{23} = (p_2 + p_3)^2 = (p - p_1)^2$. The three invariants s_{12}, s_{13} and s_{23} are not independent, it follows from their definitions together with 4-momentum conservation that $s_{12} + s_{13} + s_{23} = m^2 + m_1^2 + m_2^2 + m_3^2$. We take $s_{12} = s$ and $s_{23} = t$, so we have $s_{13} = m^2 + m_1^2 + m_2^2 + m_3^2 - s - t$. With these definitions, we obtain multiplying of the 4-momentum as: $p_1 \cdot p_3 = (1/2)(m^2 + m_2^2 - s - t)$. The decay width of a three-body process is given by

$$\Gamma(B_c^+ \to K^+K^-\pi^+) = \frac{1}{(2\pi)^3 32 m_{B_c^+}^3}\int_{s_{\min}}^{s_{\max}}\int_{t_{\min}}^{t_{\max}} |A(B_c^+ \to K^+K^-\pi^+)|^2 ds dt, \tag{2.14}$$

where $t_{\min,\max}(s) = m_1^2 + m_3^2 - (1/(2s))\Big((m^2 - s - m_3^2)(s - m_2^2 + m_1^2) \mp \sqrt{\lambda(s, m^2, m_3^2)}\sqrt{\lambda(s, m_1^2, m_2^2)}\Big)$, $s_{\min} = (m_1 + m_2)^2$, $s_{\max} = (m - m_3)^2$ and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$.

3 Numerical results and conclusion

The theoretical predictions depend on many input parameters such as Wilson coefficients, the CKM matrix elements, masses, lifetimes, decay constants, form factors, and so on. We present all the relevant input parameters as follows:

Wilson coefficients, the Wilson coefficients c_1, c_2, c_3 and c_4 in the effective weak Hamiltonian have been reliably evaluated by the next-to-leading logarithmic order. To proceed, we use the following numerical values at $\mu = m_b$ scale, which have been obtained in the NDR scheme [22]: $c_1 = 1.081$, $c_2 = -0.190$, $c_3 = 0.014$ and $c_4 = -0.036$.

The CKM matrix elements, the Cabibbo-Kobayashi-Maskawa (CKM) matrix is a 3×3 unitary matrix, the elements of this matrix can be parameterized by three mixing angles A, λ, ρ and a CP-violating phase η [17]: $V_{ud} = 1 - \lambda^2/2$, $V_{us} = \lambda$, $V_{ub} = A\lambda^3(\rho - i\eta)$, $V_{cd} = -\lambda$, $V_{cs} = 1 - \lambda^2/2$, $V_{cb} = A\lambda^2$, $V_{td} = A\lambda^3(1 - \rho - i\eta)$, $V_{ts} = -A\lambda^2$ and $V_{tb} = 1$.

JHEP12(2017)019

The results for the Wolfenstein parameters are $\lambda = 0.22537 \pm 0.00061$, $A = 0.814^{+0.023}_{-0.024}$, $\bar{\rho} = 0.117 \pm 0.021$ and $\bar{\eta} = 0.353 \pm 0.013$. We use the values of the Wolfenstein parameters and obtain $V_{ud} = 0.97427 \pm 0.00014$, $V_{us} = 0.22536 \pm 0.00061$, $V_{ub} = 0.00355 \pm 0.00015$, $V_{cd} = 0.22522 \pm 0.00061$, $V_{cs} = 0.97343 \pm 0.00015$, $V_{cb} = 0.0414 \pm 0.0012$, $V_{td} = 0.00886^{+0.00033}_{-0.00032}$, $V_{ts} = 0.0405^{+0.0011}_{-0.0012}$ and $V_{tb} = 0.99914 \pm 0.00005$.

Masses and decay constants (in units of MeV), the meson masses and decay constants needed in our calculations are taken as [17]: $m_{B_s^*} = 5415.4^{+1.8}_{-1.5}$, $m_{B_s^0} = 5366.79 \pm 0.23$, $m_{B^{*+}} = 5324.83 \pm 0.32$, $m_{\chi_{c0}} = 3414.75 \pm 0.31$, $m_{D_s^{*+}} = 2112.1 \pm 0.4$, $m_{D^0} = 1864.84 \pm 0.05$, $m_{K^{*0}} = 895.81 \pm 0.19$, $m_{K^\pm} = 493.677 \pm 0.016$, $m_{\pi^+} = 139.57018 \pm 0.00035$, $m_b = 4180 \pm 30$, $m_c = 1275 \pm 25$, $m_s = 95 \pm 5$, $m_d = 4.8^{+0.5}_{-0.3}$, $m_u = 2.3^{+0.7}_{-0.5}$, $f_{B_c^+} = 489 \pm 4$, $f_K = 159.8 \pm 1.84$, $f_{K^*} = 217 \pm 5$, $f_{K^*}^{\perp} = 185 \pm 10$, $f_\pi = 130.70 \pm 0.46$ and $\Lambda_{\rm QCD} = 225$.

Form factors, for the parameters $F_0^{P \to M}(0)$ used in transition weak form factors we take [21, 23–25]: $F_0^{B_c^+ \to B_s^0}(0) = 0.55 \pm 0.02$, $F_0^{B_c^+ \to \chi_{c0}}(0) = 0.58 \pm 0.02$, $F_0^{B_c^+ \to D^0}(0) = 0.69$, $F_0^{B_s^0 \to K^-}(0) = 0.31$ and $F_0^{D^0 \to K^-}(0) = 0.78$.

For the modes $B_c^+ \to B_s^0 \pi^+$ and $B_c^+ \to \chi_{c0} \pi^+$ we obtain the following values for the branching ratios $\mathcal{B}(B_c^+ \to B_s^0 \pi^+) = (9.82 \pm 1.05)\%$ and $\mathcal{B}(B_c^+ \to \chi_{c0} \pi^+) = (1.28 \pm 0.21) \times 10^{-3}$, these results should be compared to the experimental measurement, but only $\left(\sigma(B_c^+)/\sigma(B^+)\right) \times \mathcal{B}(B_c^+ \to B_s^0 \pi^+) = (2.37 \pm 0.31(\text{stat}) \pm 0.11(\text{syst})^{+0.17}_{-0.13}) \times 10^{-3}$ and $\left(\sigma(B_c^+)/\sigma(B^+)\right) \times \mathcal{B}(B_c^+ \to \chi_{c0} \pi^+) = (9.8^{+3.4}_{-3.0}(\text{stat}) \pm 0.8) \times 10^{-6}$ are available in the experience [7, 9]. The ratio of the production cross-sections of the B_c^+ and B^+ mesons, $\sigma(B_c^+)/\sigma(B^+)$, can be get from the measurement involving another charmonium mode, $\left(\sigma(B_c^+)/\sigma(B^+)\right) \times \mathcal{B}(B_c^+ \to J/\psi \pi^+) = (7.0 \pm 0.3) \times 10^{-6}$ obtained from ref. [26]. Using the predictions listed in ref. [27] for $\mathcal{B}(B_c^+ \to J/\psi \pi^+)$, which span the range $(0.34 \sim 2.9) \times 10^{-3}$, we obtain $\sigma(B_c^+)/\sigma(B^+) = (0.23 \sim 2.15)\%$, so the experimental branching fractions become: $\mathcal{B}(B_c^+ \to B_s^0 \pi^+) = (8.46 \sim 128.69)\%$ and $\mathcal{B}(B_c^+ \to \chi_{c0} \pi^+) = (0.30 \sim 6.09) \times 10^{-3}$ which is in very good agreement with our prediction.

For the branching fraction of the $B_c^+ \to D^0 K^+$ and $B_c^+ \to D^0 \pi^+$ decays our calculations is obtained from $\mathcal{B}(B_c^+ \to D^0 K^+) = (5.69 + 0.74) \times 10^{-7}$ and $\mathcal{B}(B_c^+ \to D^0 \pi^+) = (10.53 + 1.47) \times 10^{-7}$, as it has been obtained in [28]. The experimental result available for $(f_c/f_u) \times \mathcal{B}(B_c^+ \to D^0 K^+)$ is $(9.3^{+2.8}_{-2.5} \pm 0.6) \times 10^{-7}$ and for $(f_c/f_u) \times \mathcal{B}(B_c^+ \to D^0 \pi^+)$ is less than 3.9×10^{-7} [29], which implies f_c/f_u values in the range 0.004–0.012, the experimental branching fractions of them is obtained from 10^{-4} order in which our result is a thousand times smaller than experimental one. Maybe for this reason for the mode $B_c^+ \to D^0(\to K^- \pi^+) K^+$, no significant deviation from the background-only hypothesis is observed [9].

For the $B_s^0 \to K^+ K^-$ mode, the decay amplitude is calculated at leading power in $\Lambda_{\rm QCD}/m_b$ and at next-to-leading order in α_s using the QCD factorization approach. The calculation of the relevant hard-scattering kernels is completed. Important classes of power corrections, including "chirally-enhanced" terms and weak annihilation contributions, are estimated and included in the phenomenological analysis as given in refs. [30, 31]. Applying other contributions from the Feynman graph to this decay (such as the contribution of tree level a_2 in addition to the a_1 mentioned in the text, penguin diagrams a_3 and a_4 and weak

JHEP12(2017)019

annihilation of b_1) we obtain $\mathcal{B}(B_s^0 \to K^+K^-) = (2.16 \pm 0.35) \times 10^{-5}$, this is while its experimental value is $\mathcal{B}(B_s^0 \to K^+K^-) = (2.50 \pm 0.17) \times 10^{-5}$ [17].

For the branching ratios of the $D^0 \to K^+K^-$ and $D^0 \to K^-\pi^+$ decays we obtain $\mathcal{B}(D^0 \to K^+K^-) = (3.78 \pm 0.14) \times 10^{-3}$ and $\mathcal{B}(D^0 \to K^-\pi^-) = (4.56 \pm 0.12)\%$ where the experimental results of them are $(4.01 \pm 0.07) \times 10^{-3}$ and $(3.93 \pm 0.04)\%$ [17].

For the pure annihilation mode of the $\chi_{c0} \to K^+K^-$ theoretical calculation via factorization approach is obtained from 10^{-11} order, while the experimental value of that is from 10^{-3} order. This is a strange result because the χ_{c0} meson has very small lifetime of $10^{-23}s$. The fact is that the branching fraction of the pure annihilation decays by using the factorization approaches becomes smaller than experimental one [32, 33], from the theoretical calculation it seems that in the decay of $\chi_{c0} \to K^+K^-$, before the K^+ and K^- mesons are produced in the final states, the pare mesons such as D_s^+, D_s^- and D^0, $\bar{D}^0$ are produced in the intermediate state so the final state interaction effects are needed.

For effects of $B_c^+ \to \bar{K}^{*0}(892)K^+$ decay to the annihilation processes we get $\mathcal{B}(B_c^+ \to \bar{K}^{*0}K^+) = (9.61 \pm 0.86) \times 10^{-7}$ as has been recently predicted to be $(10.0^{+8.1}_{-4.8}) \times 10^{-7}$ [34].

For weak annihilation region the result has been measured by LHCb [9] is $R_{\text{an};KK\pi} = (8.0^{+4.4}_{-3.8}(\text{stat}) \pm 0.6(\text{syst})) \times 10^{-8}$, by taking into account the $\sigma(B_c^+)/\sigma(B^+) = (0.23 \sim 2.15)\%$ range, the measured annihilation branching ratio to be: $\mathcal{B}(B_c^+ \to K^+K^-\pi^+)_{\text{ann}} = (1.67 \sim 56.5) \times 10^{-6}$ for which is in good agreement with the result of our calculation: $\mathcal{B}(B_c^+ \to K^+K^-\pi^+)_{\text{ann}} = (10.69 \pm 1.92) \times 10^{-6}$.

Finally, by using the obtained theoretical branching ratios the results for the quasi-two-body decay of $B_c^+ \to K^+K^-\pi^+$ become from $(2.12 \pm 0.61) \times 10^{-6}$ to $(7.56 \pm 1.71) \times 10^{-6}$.

References

[1] CDF collaboration, F. Abe et al., *Observation of the B_c meson in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV*, *Phys. Rev. Lett.* **81** (1998) 2432 [hep-ex/9805034] [INSPIRE].

[2] CDF collaboration, T. Aaltonen et al., *Observation of the Decay $B_c^\pm \to J/\psi\pi^\pm$ and Measurement of the $B_c^\pm$ Mass*, *Phys. Rev. Lett.* **100** (2008) 182002 [arXiv:0712.1506] [INSPIRE].

[3] D0 collaboration, V.M. Abazov et al., *Observation of the B_c Meson in the Exclusive Decay $B_c \to J/\psi\pi$*, *Phys. Rev. Lett.* **101** (2008) 012001 [arXiv:0802.4258] [INSPIRE].

[4] LHCb collaboration, *Measurements of B_c^+ production and mass with the $B_c^+ \to J/\psi\pi^+$ decay*, *Phys. Rev. Lett.* **109** (2012) 232001 [arXiv:1209.5634] [INSPIRE].

[5] LHCb collaboration, *Observation of $B_c^+ \to J/\psi D_s^+$ and $B_c^+ \to J/\psi D_s^{*+}$ decays*, *Phys. Rev.* **D 87** (2013) 112012 [arXiv:1304.4530] [INSPIRE].

[6] LHCb collaboration, *Observation of the decay $B_c^+ \to \psi(2S)\pi^+$*, *Phys. Rev.* **D 87** (2013) 071103 [arXiv:1303.1737] [INSPIRE].

[7] LHCb collaboration, *Observation of the Decay $B_c^+ \to B_s^0\pi^+$*, *Phys. Rev. Lett.* **111** (2013) 181801 [arXiv:1308.4544] [INSPIRE].

[8] I.P. Gouz, V.V. Kiselev, A.K. Likhoded, V.I. Romanovsky and O.P. Yushchenko, *Prospects for the B_c studies at LHCb*, *Phys. Atom. Nucl.* **67** (2004) 1559 [hep-ph/0211432] [INSPIRE].

[9] LHCb collaboration, *Study of B_c^+ decays to the $K^+K^-\pi^+$ final state and evidence for the decay $B_c^+ \to \chi_{c0}\pi^+$*, *Phys. Rev.* **D 94** (2016) 091102 [arXiv:1607.06134] [INSPIRE].

[10] E.J. Eichten and C. Quigg, *Mesons with beauty and charm: Spectroscopy*, *Phys. Rev.* **D 49** (1994) 5845 [hep-ph/9402210] [INSPIRE].

[11] W.-k. Kwong and J.L. Rosner, *Masses of new particles containing b quarks*, *Phys. Rev.* **D 44** (1991) 212 [INSPIRE].

[12] S. Godfrey, *Spectroscopy of B_c mesons in the relativized quark model*, *Phys. Rev.* **D 70** (2004) 054017 [hep-ph/0406228] [INSPIRE].

[13] Quarkonium Working Group collaboration, N. Brambilla et al., *Heavy quarkonium physics*, hep-ph/0412158 [CERN-2005-005] [INSPIRE].

[14] Fermilab Lattice, HPQCD, UKQCD collaborations, I.F. Allison et al., *Mass of the B_c meson in three-flavor lattice QCD*, *Phys. Rev. Lett.* **94** (2005) 172001 [hep-lat/0411027] [INSPIRE].

[15] V.V. Kiselev, *Decays of the B_c meson*, hep-ph/0308214 [INSPIRE].

[16] LHCb collaboration, *Measurement of the lifetime of the B_c^+ meson using the $B_c^+ \to J/\psi\pi^+$ decay mode*, *Phys. Lett.* **B 742** (2015) 29 [arXiv:1411.6899] [INSPIRE].

[17] Particle Data Group collaboration, K.A. Olive et al., *Review of Particle Physics*, *Chin. Phys.* **C 38** (2014) 090001 [INSPIRE].

[18] BaBar collaboration, J.P. Lees et al., *Observation of the rare decay $B^+ \to K^+\pi^0\pi^0$ and measurement of the quasi-two body contributions $B^+ \to K^*(892)^+\pi^0$, $B^+ \to f_0(980)K^+$ and $B^+ \to \chi_{c0}K^+$*, *Phys. Rev.* **D 84** (2011) 092007 [arXiv:1109.0143] [INSPIRE].

[19] H.-Y. Cheng, *Hadronic charmed meson decays involving axial vector mesons*, *Phys. Rev.* **D 67** (2003) 094007 [hep-ph/0301198] [INSPIRE].

[20] H.-Y. Cheng, C.-K. Chua and C.-W. Hwang, *Covariant light front approach for s wave and p wave mesons: Its application to decay constants and form-factors*, *Phys. Rev.* **D 69** (2004) 074025 [hep-ph/0310359] [INSPIRE].

[21] R. Dhir and R.C. Verma, *B_c Meson Form-factors and $B_c \to PV$ Decays Involving Flavor Dependence of Transverse Quark Momentum*, *Phys. Rev.* **D 79** (2009) 034004 [arXiv:0810.4284] [INSPIRE].

[22] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, *QCD factorization in $B \to \pi K, \pi\pi$ decays and extraction of Wolfenstein parameters*, *Nucl. Phys.* **B 606** (2001) 245 [hep-ph/0104110] [INSPIRE].

[23] H.-M. Choi and C.-R. Ji, *Non-leptonic two-body decays of the B_c meson in light-front quark model and QCD factorization approach*, *Phys. Rev.* **D 80** (2009) 114003 [arXiv:0909.5028] [INSPIRE].

[24] M.A. Ivanov, J.G. Korner and P. Santorelli, *The semileptonic decays of the B_c meson*, *Phys. Rev.* **D 63** (2001) 074010 [hep-ph/0007169] [INSPIRE].

JHEP12(2017)019

[25] D. Melikhov and B. Stech, *Weak form-factors for heavy meson decays: An Update*, *Phys. Rev.* **D 62** (2000) 014006 [hep-ph/0001113] [INSPIRE].

[26] LHCb collaboration, *Measurement of B_c^+ production in proton-proton collisions at $\sqrt{s} = 8$ TeV*, *Phys. Rev. Lett.* **114** (2015) 132001 [arXiv:1411.2943] [INSPIRE].

[27] Z. Rui and Z.-T. Zou, *S-wave ground state charmonium decays of B_c mesons in the perturbative QCD approach*, *Phys. Rev.* **D 90** (2014) 114030 [arXiv:1407.5550] [INSPIRE].

[28] H.-F. Fu, Y. Jiang, C.S. Kim and G.-L. Wang, *Probing Non-leptonic Two-body Decays of B_c meson*, *JHEP* **06** (2011) 015 [arXiv:1102.5399] [INSPIRE].

[29] LHCb collaboration, *Observation of $B_c^+ \to D^0 K^+$ decays*, *Phys. Rev. Lett.* **118** (2017) 111803 [arXiv:1701.01856] [INSPIRE].

[30] M. Beneke and M. Neubert, *QCD factorization for $B \to PP$ and $B \to PV$ decays*, *Nucl. Phys.* **B 675** (2003) 333 [hep-ph/0308039] [INSPIRE].

[31] J.-J. Wang, D.-T. Lin, W. Sun, Z.-J. Ji, S. Cheng and Z.-J. Xiao, *$\bar{B}_s^0 \to K\pi, KK$ decays and effects of the next-to-leading order contributions*, *Phys. Rev.* **D 89** (2014) 074046 [arXiv:1402.6912] [INSPIRE].

[32] B. Mohammadi and H. Mehraban, *Final state interaction in $B^0 \to D^0 \bar{D}^0$*, *JHEP* **07** (2011) 089 [INSPIRE].

[33] B. Mohammadi and H. Mehraban, *Final state interaction in $B^0 \to D_s^+ D_s^-$*, *J. Phys.* **G 39** (2012) 045004 [INSPIRE].

[34] Z.-J. Xiao and X. Liu, *The two-body hadronic decays of B_c meson in the perturbative QCD approach: A short review*, *Chin. Sci. Bull.* **59** (2014) 3748 [arXiv:1401.0151] [INSPIRE].

JHEP12(2017)019