Optimum portfolio selection using a hybrid genetic algorithm and analytic hierarchy process

Maghsoud Solimanpur
Faculty of Engineering, Urmia University, Urmia, Iran, and
Gholamreza Mansourfar and Farzad Ghayour
Faculty of Economics and Management, Urmia University, Urmia, Iran

Received 7 August 2012
Revised 29 December 2013
Accepted 27 January 2014

Abstract

Purpose – The purpose of this paper is to present a multi-objective model to the optimum portfolio selection using genetic algorithm and analytic hierarchy process (AHP). Portfolio selection is a multi-objective decision-making problem in financial management.

Design/methodology/approach – The proposed approach solves the problem in two stages. In the first stage, the portfolio selection problem is formulated as a zero-one mathematical programming model to optimize two objectives, namely, return and risk. A genetic algorithm (GA) with multiple fitness functions called as Multiple Fitness Functions Genetic Algorithm is applied to solve the formulated model. The proposed GA results in several non-dominated portfolios being in the Pareto (efficient) frontier. A decision-making approach based on AHP is then used in the second stage to select the portfolio from among the solutions obtained by GA which satisfies a decision-maker's interests at most.

Findings – The proposed decision-making system enables an investor to find a portfolio which suits for his/her expectations at most. The main advantage of the proposed method is to provide prima-facie information about the optimal portfolios lying on the efficient frontier and thus helps investors to decide the appropriate investment alternatives.

Originality/value – The value of the paper is due to its comprehensiveness in which seven criteria are taken into account in the selection of a portfolio including return, risk, beta ratio, liquidity ratio, reward to variability ratio, Treynor's ratio and Jensen's alpha.

Keywords Genetic algorithm, Analytic hierarchy process, Portfolio selection

Paper type Research paper

1. Introduction

Portfolio selection has been one of the most important research fields in modern finance. It seeks a best allocation of wealth among a basket of securities. In the 1950s, a well-defined theoretical structure for stock market analysis, the portfolio analysis, started up. Modern portfolio theory is based on the pioneering works of Markowitz (1952, 1959) and Sharpe (1966). Markowitz (1952) defined the mean-variance framework for portfolio construction, and it is still the most popular method. His technique selects a portfolio that minimizes risk for a given level of reward, where reward is the mean of the portfolio return distribution and risk is measured as the variance of the portfolio return distribution.

JEL classification – G11

Studies in Economics and Finance
Vol. 32 No. 3, 2015
pp. 379-394

1086-7376
DOI 10.1108/SEF-08-2012-0085

The selected portfolios with Markowitz's approach are optimum according to their risk and returns. Every possible asset combination can be plotted in risk-return space, and the collection of all such possible portfolios defines a region in this space. The line along the upper edge of this region is known as the efficient frontier (sometimes "the Markowitz frontier"). Combinations along this line represent portfolios (explicitly excluding the risk-free alternative) for which there is lowest risk for a given level of return. Conversely, for a given amount of risk, the portfolio lying on the efficient frontier represents the combination offering the best possible return. Mathematically, the efficient frontier is the intersection of the set of portfolios with minimum variance and the set of portfolios with maximum return. After Markowitz's work, researchers have shown great enthusiasm in portfolio management, trying different mathematical approaches to develop the theory of portfolio selection (Ehrgott *et al.*, 2004). Traditionally, many research works have focused on extending Markowitz's mean-variance models (Merton, 1972) and on developing new mathematical approaches to solve the problem of computation (Perold, 1984).

The increased availability of computing power in the past two decades has been used to develop new techniques of computation and optimization. Today's computational capacity and the widespread availability of computers have enabled development of a new generation of intelligent computing techniques, such as expert systems, fuzzy logic, neural networks and genetic algorithms (GAs). These "intelligent" computing techniques are concerned with performing actions that imitate human decision-makers. These methods can find solutions to problems unsolvable by traditional methods. Of these, GAs are the most popular form of evolutionary algorithms. A GA is a problem-solving technique inspired by the biological evolution. It is based on an artificially simulated process of natural selection, or survival of the fittest, known as Darwinian evolution. GAs have emerged as a powerful general-purpose search and optimization technique and have found applications in a wide range of applications. Due to multiple objective functions in portfolio selection, GAs can only identify the portfolios being in the efficient frontier. GA has been used in many recent works to find the optimum portfolios (Gilaninia *et al.*, 2011, Chen *et al.*, 2010 and Soleimani *et al.*, 2009). These have shown that different approaches of GAs are applicable and reliable in real markets to solve the portfolio optimization problem. This study uses the Multiple Fitness Functions Genetic Algorithm (MFFGA) proposed by Solimanpur *et al.* (2004) to find the best portfolios being in the efficient frontier. The reasons for using this method are:

- MFFGA considers a systematic uniform design-based approach to set the weights of objectives.
- MFFGA applies multi-directional search to find more points distributed along the Pareto-optimal frontier.
- The mathematical background of MFFGA is relatively comprehensive; therefore, it can be used in many fields including finance and particularly in portfolio optimization problems.

Any portfolio obtained by GA in the Pareto-efficient frontier has its own risk, return, liquidity, etc. It is a very controversial and tough task for an investor to select a portfolio from among the portfolios obtained by GA which satisfies his/her interests at most. This is because of the fact that the measures of interest in portfolio selection are in conflict, and it is impossible to find a portfolio which is optimum in terms of return, risk, liquidity, etc. Therefore, the AHP approach is linked to GA in this paper to select the optimum portfolio considering all criteria from among the portfolios obtained by GA. An introduction is provided about AHP in the following lines.

Multi-criteria decision-making (MCDM) attempts to find the best alternative from among the feasible alternatives in the presence of multiple, usually conflicting, decision criteria (Pomerol and Barba Romero, 2000). The MCDM techniques generally enable decision-maker to structure the problem clearly and systematically. Approaches like priority-based, outranking, distance-based and mixed methods can be considered as the primary classes of the current methods (Saaty, 1980). One of the most popular MCDM approaches is the analytic hierarchy process (AHP) (Saaty, 1980; Saaty and Vargas, 2001), which has its roots on obtaining the relative weights among the factors and the total values of each alternative based on these weights. The AHP, first introduced by Saaty (1980), is described by Partovi and Burton (1992) as "a decision-aiding tool for dealing with complex, unstructured and multi-attribute decisions". Nydick and Hill (1992) describe AHP as:

> […] a methodology to rank alternative courses of action based on the decision-maker's judgment concerning the importance of the criteria and the extent to which they are met by each alternative.

Muralidhar *et al.* (1990) support the belief that the AHP caters specifically for decision-making with multiple criteria. Apart from this, the high precision of relative priorities in the calculations enhances the effectiveness of this technique (Fong and Choi, 2000). It has been also documented that combining the AHP with the portfolio selection problem provides both ranking and weighting information (Rahmani *et al.*, 2012; Ghazanfar Ahari *et al.*, 2011; Chaouz and Ramik, 2010; Tiryaki and Ahlatcioglu, 2009).

In portfolio selection, low risk, suitable return, high liquidity, etc., are the measures for investors to judge between portfolios. In the methodology proposed in this paper, a GA is initially used to determine the set of portfolios in the efficient frontier. In addition, a decision-making technique based on AHP is then proposed to calculate weight of each criterion and rank portfolios. The portfolio with maximum rank is finally selected as the optimum portfolio.

The remainder of this paper is structured as follows. In Section 2, the portfolio selection problem is formulated as a multi-objective mathematical programming model with binary variables followed by a theoretical discussion about multi-objective optimization and uniform design matrix in Section 3. Section 4 proposes a GA with multiple fitness functions for solving the formulated model. In Section 5, the proposed GA is applied for real data adopted from the US stock market and the efficient portfolios obtained by GA are presented. Section 6 proposes a decision-making technique based on AHP and discusses the decision hierarchy and relevant criteria in portfolio selection. In Section 7, the proposed AHP is applied to obtain the optimum portfolio from among the solutions obtained by GA. Section 8 includes discussions and concluding remarks.

2. Mathematical model

2.1 Notations

The following notations are used to formulate portfolio selection problem:

- i Index for stocks;
- N total number of stocks;
- r_i return of stock i;

- R_p return of portfolio;
- σ_p risk of portfolio; and
- x_i percentage of stock i in portfolio.

2.2 Objective functions

The attempted mathematical model includes two objective functions, namely, return and risk of portfolio. These objective functions are formulated in the following:

- *Return of portfolio*: Return of portfolio is defined as the weighted average of the returns of stocks being in the portfolio and is formulated as:

$$R_p = \sum_{i=1}^{N} r_i x_i.$$

- *Risk of portfolio*: Risk of a stock is defined as a tolerance of stock's return from the mean. Risk of a portfolio is expressed as:

$$\sigma_P = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \, \mathrm{cov}_{ij}}.$$

Where cov_{ij} is the covariance between returns of stock i and j expressed as $\mathrm{cov}_{ij} = r_{ij}\sigma_i\sigma_j$. The parameter σ_i^2 is the variance of rate of return for stock i.

2.3 Constraints

Constraints of the attempted model are as follows:

$$\sum_{i=1}^{N} x_i = 1$$
$$x_i \geq 0 \, i = 1, 2, \ldots, N$$

It is worth noting that the non-negativity of decision variables is used to avoid short-selling.

3. Theoretical background

3.1 Non-dominated solution and search vectors

The objective space of a multi-objective minimization problem can be mathematically defined as follows:

$$Z = \{z \in R^q \,|\, z_1 = f_1(x),\ z_2 = f_2(x),\ \ldots,\ z_q = f_q(x)\,;\ x \in S\}.$$

Where x is a decision variable, S represents the set of feasible solutions, and q is the number of objectives. A point $z^0 \in Z$ is called a non-dominated solution if and only if there does not exist a point $z \in Z$ such that:

$$z < z_k^0 \quad \textit{for some} \quad k \in \{1, 2, \ldots, q\}$$
$$z \leq z_l^0 \quad \textit{for all} \quad l \neq k.$$

In other words, a point z^0 is called a non-dominated solution if and only if there is no other solution in the feasible solution space better than z^0 in terms of all the objectives. A weighting vector is mainly used to search the objective space. When there is only one weighting vector, the objective space is searched in only one direction. This direction causes the computation process to result in a single solution. However, in the case of multi-objective optimization, it is almost impossible to find a solution which is optimum considering all the objective functions. Therefore, MFFGA has been adopted from a study by Solimanpur *et al.* (2004) in this paper to find more points distributed throughout the efficient frontier.

The next issue in a multi-directional search is to find search vectors. In the literature, mostly user-defined or randomly generated vectors are used to search the solution space. In this paper, however, a uniform design method is used to construct uniformly directed vectors.

3.2 Uniform design

The main attempt in uniform design is to sample a small set of points from a given large set of points. Consider a space with L variables and K possible values for each variable. Then there are K^L points (combinations) in this space. The uniform design selects K points out of K^L points such that the selected points are scattered uniformly over the space of K^L points. The selected K points are denoted by a uniform matrix, $U = [u_{ij}]_{K \times L}$, where u_{ij} is the value of variable j at point i. It can be shown (Yiu-Wing and Yuping, 2000) that when K is prime and $K > L$, u_{ij} is given by:

$$u_{ij} = (i\,\sigma^{j-1} \bmod K) + 1.$$

Where σ is a parameter given in Table I. Here, L is considered as the number of objective functions and K as the number of search directions (fitness functions).

For example, for two objective functions and five search directions ($K = 5$) the parameter σ is 2. In this case, the uniform matrix U is obtained by this equation as follows:

$$U = \begin{bmatrix} 2 & 3 \\ 3 & 5 \\ 4 & 2 \\ 5 & 4 \\ 1 & 1 \end{bmatrix}.$$

4. Proposed genetic algorithm

4.1 Portfolio display

In the proposed GA any portfolio is represented by $N \times num\_bits$ genes in which N is the number of companies and *num_bits* is the number of binary bits used for representing the share of each company in the portfolio.

The share of company i in the portfolio can be calculated by:

$$x_i = \frac{v_i}{\sum_{i=1}^{N} v_i}.$$

No. of search directions	No. of objective functions	σ
5	2-4	2
7	2-6	3
11	2-10	7
13	2	5
	3	4
	4-12	6
17	2-16	10
19	2-3	8
	4-18	14
23	2, 13-14, 20-22	7
	8-12	15
	3-7, 15-19	17
29	2	12
	3	9
	4-7	16
	8-12, 16-24	8
	13-15	14
	25-28	18

Source: Yiu-Wing and Yuping (2000)

Table I. Values of σ for different number of search directions and objective functions

where v_i is the decimal value of the binary code dedicated for company i. This formula ensures that after doing genetic operations, constraints of the mathematical model will be satisfied.

For example, suppose there are three companies in the stock market and five bits are used for representing the binary code of each company. Thus, the length of each chromosome will be 15 genes. For instance, the chromosome 101111010001101 shows a typical portfolio for this case. The first five genes represent Company 1, the second five genes represent Company 2 and the last five genes stand for Company 3. The decimal value of each company is as follows:

$$\begin{aligned} v_1 &= (10111)_{10} \Rightarrow v_1 = 23 \\ v_2 &= (10100)_{10} \Rightarrow v_2 = 20. \\ v_3 &= (01101)_{10} \Rightarrow v_3 = 13 \end{aligned}$$

Therefore, the share of each company in this portfolio is obtained as follows:

$$\begin{aligned} x_1 &= 23/(23 + 20 + 13) = 0.4107 \\ x_2 &= 20/(23 + 20 + 13) = 0.3571. \\ x_3 &= 13/(23 + 20 + 13) = 0.2322 \end{aligned}$$

4.2 Fitness functions

Fitness of any chromosome in the proposed GA is calculated in K directions. Fitness of chromosome S in direction k is computed by:

$$fit_k(S) = w_{k1}R'_p(S) + w_{k2}\sigma'_p(S),$$

where:

$fit_k(S)$ is the fitness value of chromosome S in direction k;

$R'_p(S)$ is the normalized value of return of chromosome S;

$\sigma'_p(S)$ is the normalized value of risk of chromosome S;

w_{k1} is the weight of return in direction k; and

w_{k2} is the weight of risk in direction k.

The weight of objective function l in direction k can be obtained by:

$$w_{kl} = \frac{u_{kl}}{\sum_{i=1}^{l} u_{ki}}; k = 1, 2, \ldots, K \text{ and } l = 1, 2.$$

As shown in Section 3.2, for two objective functions and five search directions, the weight of each objective in each direction is obtained as follows:

$$W = \begin{bmatrix} 0.400 & 0.600 \\ 0.375 & 0.625 \\ 0.667 & 0.333 \\ 0.555 & 0.445 \\ 0.500 & 0.500 \end{bmatrix}.$$

The normalized values of return and risk of portfolio S in the current population Ω are defined as follows:

$$R'_p(S) = \frac{R_S}{\max_{U \in \Omega}(R_U)}$$

$$\sigma'_S = \frac{\min_{U \in \Omega}(\sigma_U)}{\sigma_S}$$

As seen, these normalizations convert return and risk of a portfolio into a value between 0 and 1. Specifically, the portfolio whose return is highest in the current population is normalized to 1 and those with lower return are proportionally normalized to a value less than 1. Similarly, the portfolio with minimum risk is normalized to 1 and those with higher risk are proportionally normalized to a value less than 1. Thus, a higher fitness value for a chromosome indicates that the corresponding portfolio is better from return and risk viewpoints in total.

4.3 Genetic operations

In this research, a simple single-point crossover operator is used to generate offspring from parent chromosomes. A gene of a chromosome is selected for mutation with a probability of p_m. If a gene is selected, its value is changed from 1 to 0 and vice versa.

4.4 Selection

In the proposed GA, selection probability of chromosome S in direction k is proportional to the quality of this chromosome in direction k, i.e. $fit_k(S)$. In Other words, the higher is the fitness of a chromosome, the higher should be selection probability. Therefore, the selection probability of chromosome S in direction k can be expressed as follows:

$$p_k(S) = \frac{fit_k(S)}{\sum_{\forall U} fit_k(U)}$$

5. Results obtained by the proposed GA

The GA proposed in Section 5 was programmed in MATLAB and applied for the data collected from Datastream, which includes 94 firms of S&P100 index of the US stock market (six firms are excluded from S&P100 companies due to the lack of data). The data adopted from this market pertain to period 2001-2010.

Using an MFFGA approach to solve the Markowitz' portfolio optimization problem, 12 non-dominated portfolios are obtained. Figure 1 illustrates the generated efficient frontier by the proposed GA. Overall, the annual return of optimal portfolios varies between 6.75 and 9.15 per cent, while their risk changes between 7.79 and 9.35 per cent.

A decision-making technique based on AHP is proposed in the next section which helps decision-maker to select the most suitable portfolio from among 12 portfolios.

6. Portfolio selection via AHP

As mentioned earlier, application of GA leads to several portfolios on the efficient frontier of which the optimum one should be selected. On the other hand, optimum portfolio depends on the preferences of investor. In other words, a portfolio which is optimum for an investor may

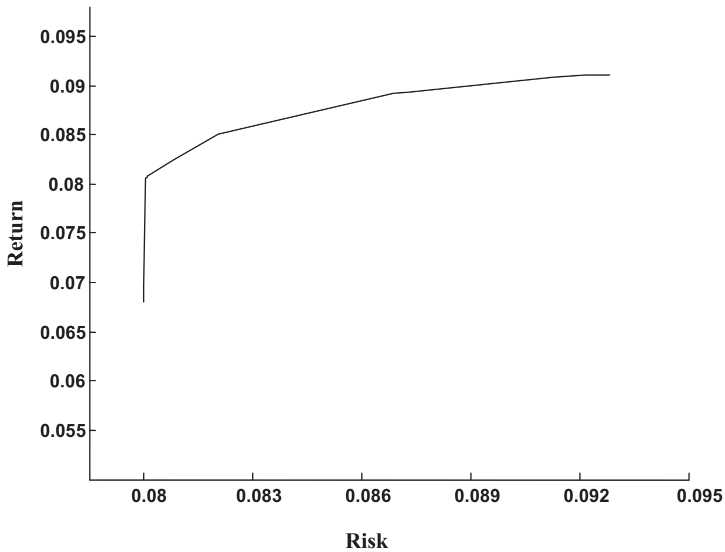

Figure 1.
Generated efficient frontier by the proposed GA

not be of interest for one another. Therefore, the evaluation procedure of the proposed method consists of the following steps (IsIklar and Büyükzkan, 2007):

- *Step 1*: Identify the set of criteria to be considered in portfolio selection (evaluation) and build a decision-making hierarchy.
- *Step 2*: Calculate weights of criteria using AHP method.

These steps are elaborated in the following subsections.

6.1 Evaluation criteria

Selecting proper portfolio performance measures that provide the necessary information for investors to assess how effectively their money may be invested is a vital issue. The performance evaluation primarily refers to the determination of how a particular investment portfolio has performed relative to some comparison benchmark. To manage portfolio selection efficiently, it is necessary to take into account the crucial factors that reflect the investors' behavior and financial market's condition. The various stakeholders within a decision process might be relatively diverse, having different objectives and conflicting value systems. In this regards, a key concept is the relationship between risk and returns. Each performance index offers a different perspective about the trade-off between return level and risk exposure; therefore, in this paper, seven measures, viz., return, risk, beta ratio, liquidity ratio, reward-to-variability ratio (RVAR), Treynor's Ratio (TR) and Jensen's alpha (Alpha ratio) have been identified as criteria which affect decision of investors in portfolio selection. Return and risk were discussed in Section 2. Other criteria are discussed in the following subsections.

6.1.1 Beta ratio. According to capital asset pricing model, there is a direct association between the systematic risk (beta) and returns where beta is the volatility of an asset relative to a market portfolio of risky assets (Chyi *et al.*, 2008; Reilly and Brown, 2006). In particular, beta ratio measures the volatility of the price of a stock relative to the rest of the market.

For example, if beta ratio of a share is −0.5, it means that if market return decreases by 20 per cent, return of this share increases by 10 per cent on average. This ratio is defined as:

$$\beta_i = \frac{Cov_{i,M}}{Var(R_M)}.$$

where:

$Cov_{i,M}$ is the covariance between the return of share i and the return of market; and

R_M is the return of market.

6.1.2 Liquidity ratio. One of the important criteria in the evaluation of a share is its liquidity. This criterion indicates the power of changing a share into a cash value. The method used in this paper for calculating the liquidity of a portfolio is as follows. Liquidity of a share is calculated by the proportion of the transactions on a specific share in a year divided by the number of company's stock. This proportion indicates annual liquidity of a share. Liquidity of a portfolio can then be calculated by:

$$L_p = \sum_{i=1}^{N} l_i x_i.$$

Where l_i is the liquidity of share i and x_i is the percentage of share i in portfolio.

6.1.3 Reward-to-variability ratio. RVAR or Sharpe ratio measures the return into the portfolio risk (standard deviation of return) (Sharpe, 1966). This ratio is defined by:

$$RVAR = \frac{\overline{TR_p} - \overline{RF}}{SD_p},$$

where:

$\overline{TR_p}$ is the mean of portfolio return in a certain period of time;

$\overline{RF}$ is the mean of non-risk return rate in time set;

SD_p is the standard deviation of portfolio return in time set; and

$\overline{TR_p} - \overline{RF}$ is the surplus return of portfolio.

This ratio will measure the surplus return of portfolio versus a risk unit. The higher is the RVAR of a portfolio, the higher will be its desirability.

6.1.4 Treynor's ratio. TR measures the proportion of extra return on beta ratio (Treynor, 1965). This ratio is defined by:

$$TR = \frac{\overline{TR_p} - \overline{RF}}{\beta_p}.$$

Where β_p is the systematic risk of portfolio (beta ratio). This ratio measures the surplus return of portfolio versus portfolio's systematic risk unit. Similar to RVAR, the greater is the TR of a portfolio, the higher will be its desirability.

It is worth highlighting the difference between RVAR and TR. If a portfolio is diverse to a major extent, both ratios will provide the same measure. However, if a portfolio is not sufficiently diverse, the value of TR ratio will be bigger than that of RVAR ratio. Therefore, investors who put all of their assets in a portfolio should more rely on RVAR ratio because the Sharp ratio will measure the portfolio's return based on the total risk including systematic and unsystematic risks. But those whose portfolio is a small part of their assets may rely on TR ratio, as it measures systematic risk only.

6.1.5 Alpha ratio. Alpha ratio is correlated to Treynor's ratio and, hence, these ratios provide a close ranking about the performance of a portfolio (Jensen, 1968). This ratio is defined as:

$$\overline{\alpha_p} = \overline{R_p} - [\overline{RF} + (R_M - \overline{RF})\beta_p].$$

The weakness and strength of a portfolio's performance has two sources. First is the capability of portfolio manager in selecting suitable shares and the second is his/her capability in making suitable decisions over time and evaluating threats and opportunities of the market. Obviously, the manager who considers these aspects will have a better performance in managing the portfolio. The benefit of using this ratio is the possibility to measure $\overline{\alpha_P}$ and β_P at the same time.

6.2 The AHP methodology

AHP includes six basic steps, as summarized in the following (IsIklar and Büyükzkan, 2007; Saaty, 1980):

1. *Step 1*: AHP breaks down a complex MCDM problem to several small sub-problems each with a single criterion. Thus, the first step is to decompose the decision problem into a hierarchy with a goal at the top, criteria and sub-criteria at levels and sub-levels and decision alternatives at the bottom of the hierarchy.
2. *Step 2*: The decision matrix, which is based on Saaty's nine-point scale, is constructed. The decision-maker uses the fundamental 1-9 scale defined by Saaty to assess the priority score. In this context, the Value 1 indicates equal importance, 3 moderately more, 5 strongly more, 7 very strongly and 9 indicates extremely more importance (Table II).

The Values 2, 4, 6 and 8 are allotted to indicate compromise values of importance. The decision matrix involves the assessments of each alternative with respect to the decision criteria. If the decision-making problem consists of n criteria and m alternatives, the decision matrix takes the form:

$$\mathbf{D} = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ d_{m1} & d_{m2} & \dots & d_{mn} \end{bmatrix}$$

The entry d_{ij} in matrix D indicates rating of the i th alternative with respect to the j th criterion.

3. *Step 3*: The third step involves pairwise comparison of the elements of the constructed hierarchy. The aim is to set their relative priorities with respect to each of the elements at the next higher level. The pairwise comparison matrix, which is based on the Saaty's 1-9 scale, has the following form:

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} = \begin{bmatrix} w_1/w_1 & w_1/w_2 & \dots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \dots & w_2/w_n \\ \vdots & \vdots & \vdots & \vdots \\ w_n/w_1 & w_n/w_2 & \dots & w_n/w_n \end{bmatrix}$$

If $n(n-1)/2$ comparisons are consistent, the elements $\{a_{ij}\}$ will satisfy the following conditions: $a_{ij} = w_i/w_j = 1/a_{ji}$ and $a_{ii} = 1$ for $i,j = 1,2 \ldots,n$.

In the comparison matrix, a_{ij} can be interpreted as the degree of preference of i th criterion over j th criterion.

Numerical assessment	Linguistic meaning
1	Equally important
3	Moderately more important
5	Strongly more important
7	Very strongly important
9	Extremely more important
2,4,6,8	Intermediate values of importance

Table II. The numerical assessments and their linguistic meanings

4. *Step 4*: AHP also calculates an inconsistency index to reflect the consistency of decision-maker's judgments during the evaluation phase. The inconsistency index in both the decision matrix and in pairwise comparison matrices can be calculated with the equation:

$$CI = \frac{\lambda_{\max} - N}{N - 1}.$$

The closer is the inconsistency index to zero; the greater will be the consistency of decision-maker's judgments. The consistency of the assessments is ensured if the equality $a_{ij} . a_{jk} = a_{ik}$ holds for all criteria. The relevant index should be lower than 0.10 to accept the AHP results as consistent. If this is not the case, the decision-maker should go back to Steps 2 and 3 and redo the assessments and comparisons.

5. *Step 5*: Before any calculation, the comparison matrix has to be normalized. To normalize, each column is divided by the sum of entries of the corresponding column. In that way, a normalized matrix is obtained in which the sum of the elements of each column is 1.

6. *Step 6*: The relative values obtained in the third step should satisfy:

$$\mathbf{AW} = \lambda_{\max} \cdot \mathbf{W},$$

Where A represents the pairwise comparison matrix, and $\lambda_{\max}$ is the highest eigenvalue. If there are elements at the higher levels of the hierarchy, the obtained weight vector is multiplied by the weights of the elements at the higher levels, until the top of the hierarchy is reached. The alternative with the highest weight is finally considered as the best alternative.

7. Computational results and discussions

The proposed hierarchical structure of the optimum portfolio selection problem along with the alternatives obtained by MFFGA and the identified criteria are depicted in Figure 2.

As seen, the decision hierarchy consists of three levels. The optimum portfolio selection is the prime objective of the problem and takes place at the topmost level (Level 1) of the hierarchy. The seven criteria, viz., return, risk, beta ratio, liquidity, RVAR, TR and alpha ratio take place at the second level. Finally, the 12 portfolios identified by MFFGA take place at the most bottom level (Level 3) as decision alternatives.

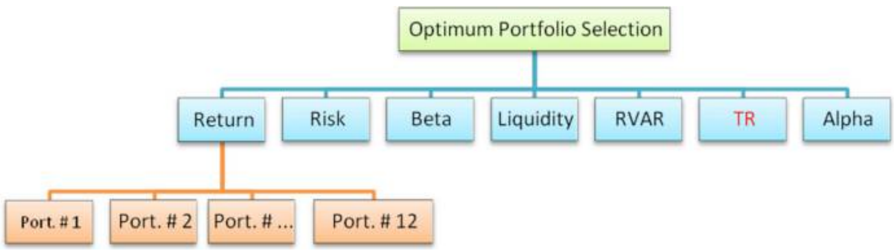

Figure 2.
Proposed hierarchy of the portfolio selection problem

As mentioned earlier, the AHP technique needs a pairwise comparison of the seven criteria to determine their weights. This matrix is a key input data which directly reflect expectations of an investor. For instance, let us assume that Table III shows pairwise comparison of seven criteria for a typical investor.

The total weight of each criterion is shown in Table IV.

For example, the weight of return versus risk is 5. This means that return is strongly more important than risk for this investor. The normalized weights of the seven criteria and their ranking are shown in Table V. Table V indicates weight of each portfolio versus each criterion.

Table VI shows the total weight of each portfolio in terms of all criteria.

Criteria	Return	Risk	Beta	Liquidity	RVAR	TR	Alpha
Return	1	5	7	3	2	3	3
Risk	1/5	1	12/5	3/5	2/5	3/5	3/5
Beta	1/7	5/7	1	3/7	2/7	3/7	3/7
Liquidity	1/3	12/3	21/3	1	2/3	1	1
RVAR	1/2	21/2	31/2	11/2	1	11/2	11/2
TR	1/3	12/3	21/3	1	2/3	1	1
Alpha	1/3	12/3	21/3	1	2/3	1	1

Table III. The pairwise comparison matrix of the criteria

Criterion	Total weight (%)	Rank
Return	0.3518	1
Risk	0.0704	4
Beta	0.0503	5
Liquidity	0.1173	3
RVAR	0.1759	2
TR	0.1173	3
Alpha	0.1173	3

Table IV. Priority weights in the AHP decision tree

Portfolio no.	Return	Risk	Beta	Liquidity	RVAR	TR	Alpha
1	0.0675	0.0779	0.0823	0.0757	0.0670	0.0634	0.0626
2	0.0692	0.0779	0.0843	0.0825	0.0694	0.0641	0.0648
3	0.0806	0.0780	0.0817	0.0844	0.0853	0.0813	0.0797
4	0.0807	0.0780	0.0817	0.0845	0.0855	0.0815	0.0799
5	0.0808	0.0780	0.0818	0.0846	0.0856	0.0815	0.0800
6	0.0825	0.0789	0.0785	0.0747	0.0871	0.0874	0.0823
7	0.0853	0.0804	0.0843	0.0768	0.0892	0.0849	0.0859
8	0.0896	0.0863	0.0843	0.0896	0.0886	0.0905	0.0915
9	0.0897	0.0868	0.0845	0.0904	0.0883	0.0905	0.0917
10	0.0912	0.0916	0.0872	0.0848	0.0854	0.0896	0.0937
11	0.0914	0.0928	0.0847	0.0859	0.0846	0.0926	0.0940
12	0.0915	0.0935	0.0847	0.0861	0.0840	0.0926	0.0941

Table V. Weights of portfolios toward criteria

Table VI.
Final weight and rank of portfolios

Portfolio no.	Final weight	Rank
1	0.06881	12
2	0.07107	11
3	0.08171	10
4	0.08185	9
5	0.08191	8
6	0.08250	7
7	0.08461	6
8	0.08924	5
9	0.08940	3
10	0.08937	4
11	0.08978	1
12	0.08976	2

For example, to calculate final weight of portfolio 1, first row of Table V is multiplied by the second column of Table IV. As seen in Table VI, portfolio 11 has gained maximum weight of 0.08978 and, hence, possesses the first rank among the 12 portfolios obtained by GA. Therefore, portfolio 11 is selected as the final optimum solution which can serve expectations of the current investor mentioned in Table III at most.

8. Conclusions

This paper aims at providing an optimum decision-making system for selecting an optimum and suitable portfolio according to some criteria of interest for investors. The proposed decision-making system is composed of two stages. Stage 1 exploits a GA to identify a set of portfolios in the efficient frontier. The portfolios obtained by GA are non-dominated in terms of return and risk. Of these portfolios, an optimum portfolio is selected in Stage 2, using an AHP-based decision-making hierarchy. The overall findings provide new areas of exploration in portfolio selection. The main advantage of the proposed method is to provide prima-facie information about the optimal portfolios lying on the efficient frontier and, thus, helps investors to decide the appropriate investment alternatives. The investors, then, may pick and choose from among these alternatives by obtaining the desired portfolio with the help of the proposed hybrid decision-making system using MFFGA and AHP process.

The proposed method can be further enhanced and improved in future researches as discussed below:

- Optimality of the final solution largely depends on the algorithm by which alternative portfolios are identified. A GA has been proposed in this paper to identify these alternatives. Application of other meta-heuristics like tabu search, simulated annealing, scatter search, etc., can be tested.
- The proposed decision-making hierarchy includes seven criteria. This hierarchy can be further completed by adding other quantitative or qualitative criteria not captured in the proposed hierarchy.
- The proposed decision-making system can be integrated with a knowledge base and some optimization tools to build an expert system for optimum portfolio selection.

References

Charouz, J. and Ramik, J. (2010), "A multicriteria decision making at portfolio management", *Ekonomoka Management*, Vol. 2 No. 1, pp. 44-52.

Chen, Y., Mabu, S. and Hirasawa, K. (2010), "A model of portfolio optimization using time adapting genetic network programming", *Computers & Operations Research*, Vol. 37 No. 10, pp. 1697-1707.

Chyi, L.L., Robinson, J. and Reed, R. (2008), "Downside beta and the cross-sectional determinants of listed property trust returns", *Journal of Real Estate Portfolio Management*, Vol. 14 No. 1, pp. 49-62.

Ehrgott, M., Klamroth, K. and Schwehm, C. (2004), "An MCDM approach to portfolio optimization", *European Journal of Operational Research*, Vol. 155 No. 3, pp. 752-770.

Fong, P.S.W. and Choi, S.K.Y. (2000), "Final contractor selection using the analytical hierarchy process", *Construction Management and Economics*, Vol. 18 No. 5, pp. 547-557.

Ghazanfar Ahari, S., Ghaffari-Nasab, N., Makui, A. and Ghodsypour, S.H. (2011), "A portfolio selection using fuzzy analytic hierarchy process: a case study of Iranian pharmaceutical industry", *International Journal of Industrial Engineering*, Vol. 2 No. 2, pp. 225-236.

Gilaninia, S., Mousavian, S. and Rezvani, M. (2011), "Optimization of technical rules by Genetic Algorithms (GA) in IRAN stock market", *Interdisciplinary Journal of Contemporary Research in Business*, Vol. 3 No. 2, pp. 142-153.

IsIklar, G. and Büyükzkan, G. (2007), "Using a multi-criteria decision making approach to evaluate mobile phone alternatives", *Computer Standards & Interfaces*, Vol. 29 No. 2, pp. 265-274.

Jensen, M.C. (1968), "The performance of mutual funds in the period 1945-1964", *Journal of Finance*, Vol. 23 No. 2, pp. 389-416.

Markowitz, H. (1952), "Portfolio selection", *Journal of Finance*, Vol. 7 No. 1, pp. 77-91.

Markowitz, H. (1959), *Portfolio Selection: Efficient Diversification of Investments*, Wiley, NewYork, NY.

Merton, R. (1972), "An analytic derivation of the efficient portfolio frontier", *Journal of Finance and Quantitative Analysis*, Vol. 7 No. 4, pp. 1851-1872.

Muralidhar, K., Santhnam, R. and Wilson, R.L. (1990), "Using the analytic hierarchy process for information system project selection", *Information and Management*, Vol. 18 No. 2, pp. 87-90.

Nydick, R.L. and Hill, R.P. (1992), "Using the analytic hierarchy process to structure the supplier selection procedure", *International Journal of Purchasing and Materials Management Research News*, Vol. 28 No. 2, pp. 31-36.

Partovi, F.Y. and Burton, J. (1992), "An analytical hierarchy approach to facility layout", *Computers and Industrial Engineering*, Vol. 22 No. 4, pp. 447-457.

Perold, A.F. (1984), "Large-scale portfolio optimization", *Management Science*, Vol. 30 No. 10, pp. 1143-1160.

Pomerol, J.C. and Barba Romero, S. (2000), *Multicriterion Decision in Management: Principles and Practice*, Kluwer Academic Publishers, Norwell.

Rahmani, N., Talebpour, A. and Ahmadi, T. (2012), "Developing a multi criteria model for stochastic IT portfolio selection by AHP method", *Social and Behavioral Sciences*, World Conference on Business, Economics and Management (BEM-2012), Antalya, 4-6 May, Vol. 62, pp. 1041-1045.

Reilly, F.K. and Brown, K.C. (2006), *Investment Analysis and Portfolio Management*, Thomson South-Western Pub, Mason.

Saaty, T.L. (1980), *The Analytic Hierarchy Process*, McGraw Hill Company, New York, NY.

Saaty, T.L. and Vargas, L.G. (2001), *Models, Methods, Concepts & Applications of the Analytic Hierarchy Process*, Kluwer Academic, Boston, MA.

Sharpe, W.F. (1966), "Mutual fund performance", *The Journal of Business*, Vol. 39 No. 1, pp. 119-139.

Soleimani, H., Golmakani, H. and Salimi, M. (2009), "Markowitz-based portfolio selection with minimum transaction lots, cardinality constraints and regarding sector capitalization using genetic algorithm", *Expert Systems with Applications*, Vol. 36 No. 1, pp. 5058-5063.

Solimanpur, M., Vrat, P. and Shankar, R. (2004), "A multi-objective genetic algorithm approach to the design of cellular manufacturing systems", *International Journal of Production Research*, Vol. 42 No. 7, pp. 1419-1441.

Tiryaki, F. and Ahlatcioglu, B. (2009), "Fuzzy portfolio selection using fuzzy analytic hierarchy process", *Information Science*, Vol. 179 Nos 1/2, pp. 53-69.

Treynor, J.L. (1965), "How to rate management of investment funds", *Harvard Business Review*, Vol. 43 No. 1, pp. 63-75.

Yiu-Wing, L. and Yuping, W. (2000), "Multiobjective programming using uniform design and genetic algorithm", *Systems, Man, and Cybernetics, Part C: Applications and Reviews, IEEE Transactions on*, Vol. 30 No. 3, pp. 293-304.

Corresponding author
Farzad Ghayour can be contacted at: f.ghayour@urmia.ac.ir